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TE 364: Communication Circuits

Lecture 8

Filter Circuits 2

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Insertion-Loss Method

- Provides ways to shape pass- and stop-bands of the filter,
  - although its design theory is much more complex
- Power Loss Ratio, P_{LR}

$$P_{\mathrm{LR}} = \frac{\text{power incident at port 1}}{\text{power delivered to the load connected at port 2}} = \frac{1}{1 - |\Gamma(\omega)|^2}$$

  - Expressed in decibel, it is called the *insertion loss*

Insertion-Loss Method

$$\Gamma\left(\omega^2\right) = \frac{f_1\left(\omega^2\right)}{f_1\left(\omega^2\right) + f_2\left(\omega^2\right)}$$

$$P_{LR} = 1 + \frac{f_1\left(\omega^2\right)}{f_2\left(\omega^2\right)}$$

$f_1\left(\omega^2\right)$ and $f_2\left(\omega^2\right)$ are real polynomials of ω^2

Insertion-loss Method

- Magnitude of the voltage gain of the 2-port is

$$\left|G(\omega)\right| = \frac{1}{\sqrt{P_{LR}}} = \frac{1}{\sqrt{1 + f_1\left(\omega^2\right)/f_2\left(\omega^2\right)}}$$

- Design Procedure
  - Begin with lumped-element low-pass prototype
    - Synthesized from normalized tables
  - Low-pass then transformed to required cut-off and impedance.
  - Prototype transformed to high-pass, band-pass etc.
  - Lumped elements then transformed to t-lines

Maximally Flat Filter

- Flattest possible pass-band respond
- Also known as
  - Butterworth filter
  - Binomial filter

$$|G(\omega)| = \frac{1}{\sqrt{1+\zeta\bar{\omega}^{2n}}}$$

n is the order of the filter

ζ controls the power-loss ratio at its band edge

$\bar{\omega}$ is the normalized frequency

Maximally Flat Filter

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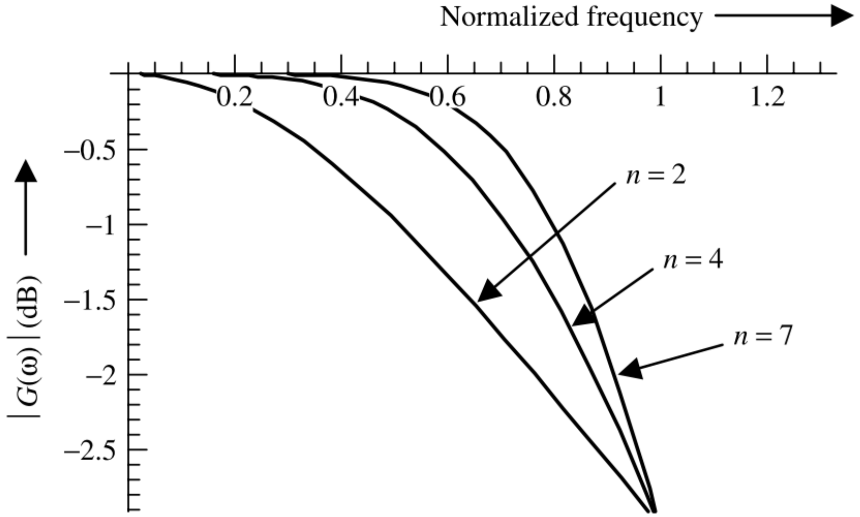
- Insertion loss is given by

$$L = -20\log_{10}\frac{1}{\sqrt{1+\zeta\left(\omega/\omega_c\right)^{2n}}} = 10\log_{10}\left[1+\zeta\left(\frac{\omega}{\omega_c}\right)^{2n}\right]$$



Maximally Flat Filter

- At the band edge, $\omega = \omega_c$

  and

$$\zeta = 10^{L_c/10} - 1$$

- The order n can be obtained from

$$n = \frac{1}{2} \times \frac{\log_{10}(10^{L/10} - 1) - \log_{10}\zeta}{\log_{10}(\omega/\omega_c)}$$

Chebyshev Filter

- For Chebyshev Filters,
  - Flat pass-band is sacrificed for sharper cut-off
- Therefore
  - Possess ripples in the pass-band
  - Have sharp transition into the stop-band
- Chebyshev polynomials are used
  - To represent the insertion loss

Chebyshev Filter

- Mathematically,

$$\left|G(\omega)\right| = \frac{1}{\sqrt{1+\zeta T_m^2\left(\overline{\omega}\right)}} \qquad m = 1, 2, 3, \ldots$$

T_m is a Chebyshev polynomial

ζ is a constant that controls the power-loss ratio at its band edge

$\overline{\omega}$ is the normalized frequency

Chebyshev Filter

- The insertion loss is given by

$$L = -20\log_{10}\frac{1}{\sqrt{1+\zeta T_m^2(\omega/\omega_c)}}$$

$$= 10\log_{10}\left[1+\zeta T_m^2\left(\frac{\omega}{\omega_c}\right)\right]$$

- Or

Chebyshev Filter

$$L = \begin{cases} 10\log_{10}\left[1+\zeta\cos^2\left(m\cos^{-1}\dfrac{\omega}{\omega_c}\right)\right] & 0 \le \omega \le \omega_c \\ 10\log_{10}\left[1+\zeta\cosh^2\left(m\cosh^{-1}\dfrac{\omega}{\omega_c}\right)\right] & \omega_c < \omega \end{cases}$$

- Where

$$\zeta = 10^{0.1\times G_r} - 1$$

- G_r is the ripple amplitude in decibel

Chebyshev Filter

$$m = \frac{\cosh^{-1}\sqrt{\left(10^{0.1\times L}-1\right)/\left(10^{0.1\times G_r}-1\right)}}{\cosh^{-1}\left(\omega/\omega_c\right)}$$

- Where
  - L is the required insertion loss at frequency ω

Chebyshev Filter

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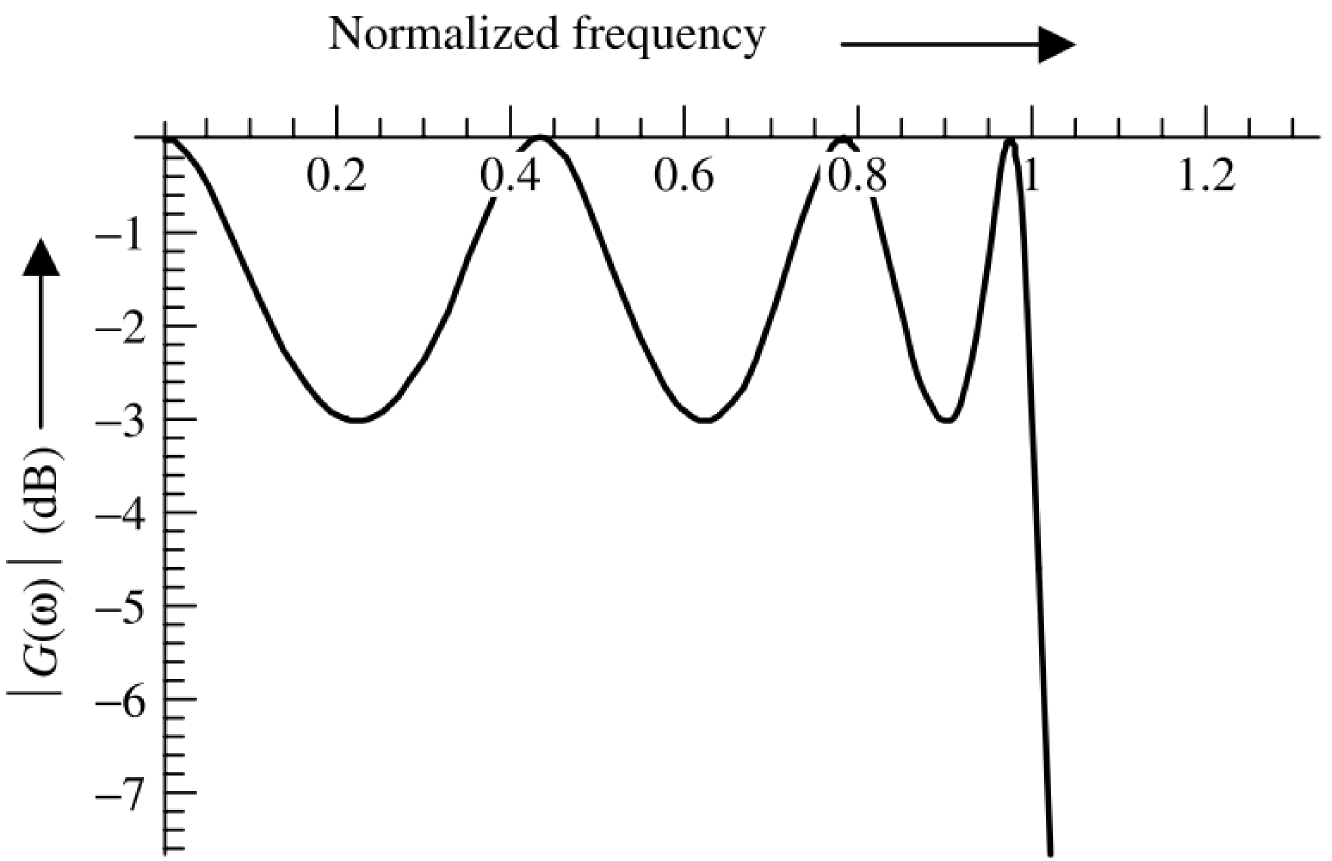

m=7

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Chebyshev Filter

Example

- It is desired to design a maximally flat low-pass filter with at least 15 dB attenuation at $\omega = 1.3\omega_c$ and -3 dB at its band edge. How many elements will be required for this filter? If a Chebyshev filter is used with a 3-dB ripple in its pass-band, find the number of circuit elements.

Solution

$$\zeta = 10^{0.1\times L_c} - 1 = 10^{0.3} - 1 = 1$$

$$n = \frac{1}{2} \times \frac{\log_{10}(10^{L/10} - 1) - \log_{10}\zeta}{\log_{10}(\omega/\omega_c)} = 0.5 \times \frac{\log_{10}(10^{1.5} - 1)}{\log_{10}(1.3)} = 6.52$$

Therefore, seven elements will be needed for this maximally flat filter.

Chebyshev Filter

- In the case of Chebyshev filter

$$m = \frac{\cosh^{-1}\sqrt{\left(10^{0.1\times L}-1\right)/\left(10^{0.1\times G_r}-1\right)}}{\cosh^{-1}\left(\omega/\omega_c\right)} = \frac{\cosh^{-1}\sqrt{\left(10^{1.5}-1\right)}}{\cosh^{-1}\left(1.3\right)} = 3.17$$

- Therefore, only 3 elements are required

Low-Pass Filter Synthesis

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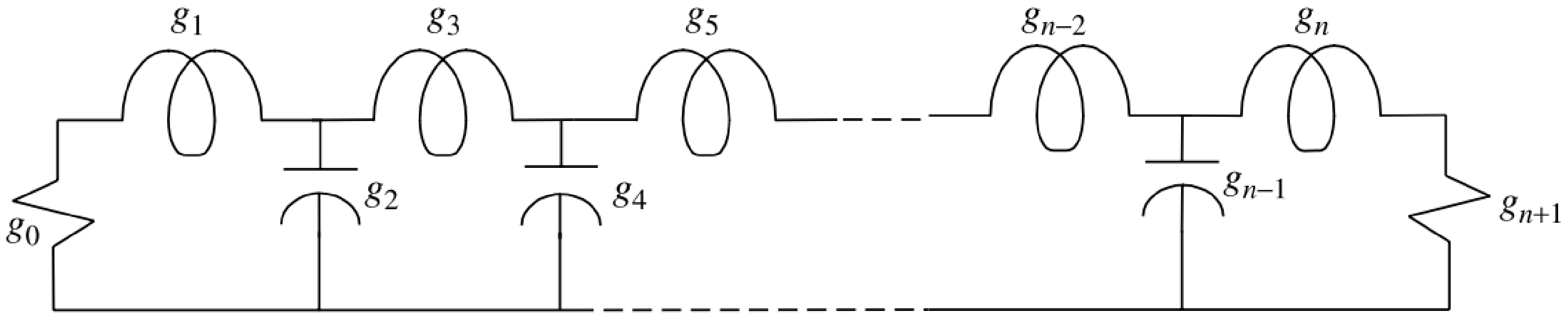
- Doubly terminated low-pass ladder network



  - g signifies the roots of an nth-order transfer function that govern the characteristics of the filter.
  - These represent the normalized reactance values of filter elements with a cut-off frequency $\omega_c = 1$.
- The source resistance is represented by g_0 , and load is g_{n+1}.

Low-Pass Filter Synthesis

- The filter is made up of
  - series inductors and shunt capacitors that are in the form of cascaded T-networks.
- Another possible configuration is
  - a cascaded π -network that is obtained after replacing g_1 by a short circuit,
  - connecting a capacitor across the load g_{n+1}, and
  - renumbering the filter elements 1 through n.
- Elements are determined from the n roots of transfer function.

Low-Pass Filter Synthesis

- The transfer function is selected according to the pass- and stop-band characteristics desired.
- Normalized values of elements are then found from the roots of that transfer function.
- These values are then adjusted according to
  - the desired cut-off frequency and
  - the source and load resistance.

Summary of Design Procedure for Maximally Flat LP Filters

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- Assume that the cut-off frequency is given as $\omega_c = 1$
- For Butterworth
  - $g_0 = g_{n+1} = 1$
  - $g_p = 2\sin\dfrac{(2p-1)\pi}{2n} \qquad p = 1, 2, 3, \ldots$
  - Such element values are given in the following table

Summary of Design Procedure for Maximally Flat LP Filters

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- Design values for Low-pass Binomial Filter Prototype

TABLE 9.4 Element Values for Low-Pass Binomial Filter Prototypes ($g_0 = 1$, $\omega_c = 1$)

n	g_1	g_2	g_3	g_4	g_5	g_6	g_7	g_8
1	2.0000	1.0000						
2	1.4142	1.4142	1					
3	1.0000	2.0000	1.0000	1.0000				
4	0.7654	1.8478	1.8478	0.7654	1.0000			
5	0.6180	1.6180	2.0000	1.6180	0.6180	1		
6	0.5176	1.4142	1.9319	1.9319	1.4142	0.5176	1	
7	0.4450	1.2470	1.8019	2.0000	1.8019	1.2470	0.445	1.0000

Summary of Design Procedure for Chebyshev LP Filters

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$$g_o = 1$$

$$g_{m+1} = \begin{cases} 1 & m \text{ is an odd number} \\ \coth \dfrac{\xi}{4} & m \text{ is even number} \end{cases}$$

$$g_1 = \frac{2a_1}{\chi}, \text{ and}$$

$$g_p = \frac{4a_{(p-1)}a_p}{b_{(p-1)}g_{(p-1)}} \qquad p = 1, 2, \cdots m$$

$$\xi = \ln\left(\coth \frac{G_r}{17.37}\right)$$

$$\chi = \sinh \frac{\xi}{2m}$$

$$a_p = \sin \frac{(2p-1)\pi}{2m}$$

$$b_p = \chi^2 + \sin^2 \frac{p\pi}{m}$$

Summary of Design Procedure for Chebyshev LP Filters

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- Design values for Low-pass Binomial Filter Prototype

TABLE 9.5 Element Values for Low-Pass Chebyshev Filter Prototypes ($g_0 = 1$, $\omega_c = 1$, and 0.1 dB ripple)

m	g_1	g_2	g_3	g_4	g_5	g_6	g_7	g_8
1	0.3053	1.000						
2	0.8431	0.6220	1.3554					
3	1.0316	1.1474	1.0316	1.0000				
4	1.1088	1.3062	1.7704	0.8181	1.3554			
5	1.1468	1.3712	1.9750	1.3712	1.1468	1.0000		
6	1.1681	1.4040	2.0562	1.5171	1.9029	0.8618	1.3554	
7	1.1812	1.4228	2.0967	1.5734	2.0967	1.4228	1.1812	1.0000

Scaling the Prototype to the Desired Cutoff Frequency and Load

- **Frequency scaling from 1 to ω_c.**
  - divide all normalized *g* values that represent capacitors or inductors by the desired cut-off frequency expressed in radians per second.
  - Resistors are excluded from this operation.
- **Impedance scaling g_0 and g_{n+1} to X Ω from unity.**
  - multiply all *g* values that represent resistors or inductors by X.
  - On other hand, divide those *g* values representing capacitors by X.

LP Butterworth example

Example

- Design a Butterworth filter with a cut-off frequency of 10 MHz and an insertion loss of 30 dB at 40 MHz. It is to be used between a 50 Ω load and a generator with an internal resistance of 50 Ω.

Solution

$$n = \frac{1}{2} \times \frac{\log_{10}(10^{30/10} - 1) - \log_{10}(10^{3/10} - 1)}{\log_{10}(40/10)}$$

$$= \frac{1}{2} \times \frac{\log_{10}(10^{3} - 1) - \log_{10}(1.9953 - 1)}{\log_{10}(4)} \approx \frac{0.5 \times 3}{0.6} \approx 2.5$$

Therefore, $n = 3$ elements will be needed for this maximally flat filter.

LP Butterworth example

$$g_0 = g_4 = 1$$

$$g_1 = 2\sin\frac{(2-1)\pi}{2\times 3} = 2\sin\frac{\pi}{6} = 1$$

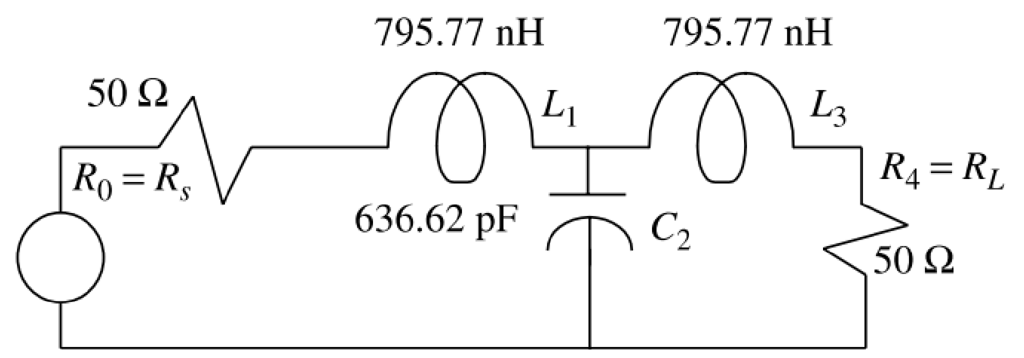
$$g_2 = 2\sin\frac{(4-1)\pi}{2\times 3} = 2\sin\frac{\pi}{2} = 2$$

$$g_3 = 2\sin\frac{(6-1)\pi}{2\times 3} = 2\sin\frac{5\pi}{6} = 1$$

scaling element values

$$L_1 = L_3 = 50 \times \frac{1}{2\pi \times 10^7}\text{H} = 795.77\,\text{nH}$$

$$C_2 = \frac{1}{50} \times \frac{2}{2\pi \times 10^7}\text{F} = 636.62\,\text{pF}$$



LP Chebyshev example

- ***Example***
  - Design a low-pass Chebyshev filter that may have ripples no more than 0.01 dB in its pass-band. The filter must pass all frequencies up to 100 MHz and attenuate the signal at 400 MHz by at least 5 dB. The load and the source resistance are of 75 Ω each.

- ***Solution***

Since $G_r = 0.01$ and $L = 5\text{dB}$

$$m = \frac{\cosh^{-1}\sqrt{(10^{0.5}-1)/(10^{0.001}-1)}}{\cosh^{-1}(4)} = 2$$

Since we want a symmetrical filter with 75 Ω on each side, we select $m = 3$.

LP Chebyshev example

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$$g_0 = g_4 = 1$$

$$a_1 = \sin\frac{(2-1)\pi}{2\times 3} = \sin\frac{\pi}{6} = 0.5$$

$$a_2 = \sin\frac{(4-1)\pi}{2\times 3} = \sin\frac{\pi}{2} = 1$$

$$a_3 = \sin\frac{(6-1)\pi}{2\times 3} = \sin\frac{5\pi}{6} = 0.5$$

$$\xi = \ln\left(\coth\frac{0.01}{17.37}\right) = 7.5$$

$$\chi = \sinh\frac{7.5}{6} = 1.6019$$

$$b_1 = 1.6019^2 + \sin^2\frac{\pi}{3} = 3.316$$

$$b_2 = 1.6019^2 + \sin^2\frac{2\pi}{3} = 3.316$$

$$b_3 = 1.6019^2 + \sin^2\frac{3\pi}{3} = 2.566$$

LP Chebyshev example

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$$g_0 = g_4 = 1$$

$$g_1 = \frac{2 \times 0.5}{1.6019} = 0.62425$$

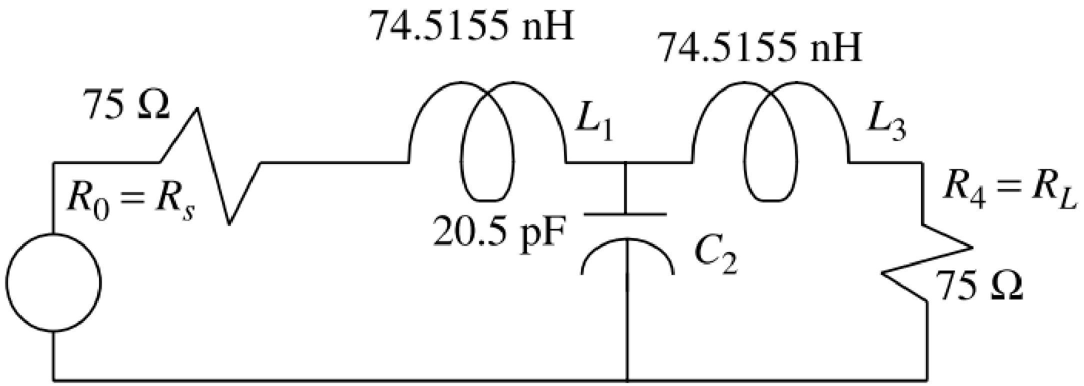
$$g_2 = \frac{4 \times 0.5 \times 1}{3.316 \times 0.62425} = 0.9662$$

$$g_3 = \frac{4 \times 1 \times 0.5}{3.316 \times 0.9662} = 0.62425$$

scaling element values

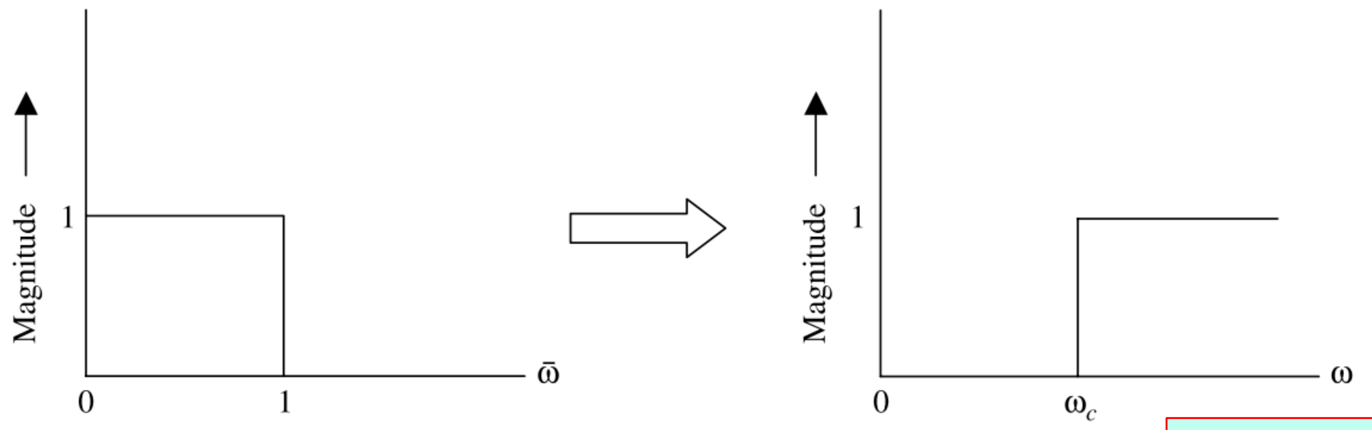
$$L_1 = L_3 = \frac{75 \times 0.62425}{2\pi \times 10^8}\,\mathrm{H} = 74.5155\,\mathrm{nH}$$

$$C_2 = \frac{1}{75} \times \frac{1}{2\pi \times 10^8} \times 0.9662\,\mathrm{F} = 20.5\,\mathrm{pF}$$



High Pass Filter

- A high-pass filter can be designed by
  - transforming the low-pass prototype
  - The frequency transformation is shown below



  - The frequency transformation is thus $\bar{\omega} = -\dfrac{\omega}{\omega_c}$

High Pass Filter

- Thus, inductors and capacitors will change their places.
  - Inductors will replace the shunt capacitors of the low-pass filter and
  - capacitors will be connected in series, in place of inductors.
- The elements are determined as follows:

$$C_{\mathrm{HP}} = \frac{1}{\omega_c g_L} \qquad L_{\mathrm{HP}} = \frac{1}{\omega_c g_C}$$

- Capacitor C_{HP} and inductor L_{HP}
  - are then scaled as required by the load and source resistance.

High Pass Filter

Example

- Design a high-pass Chebyshev filter with pass-band ripple magnitude less than 0.01 dB. It must pass all frequencies over 100 MHz and exhibit at least 5 dB of attenuation at 25 MHz. Assume that the load and source resistances are at 75 Ω each.

Solution

The low-pass filter designed in Example 9.7 provides the initial data for this high-pass filter. With $m = 3$, $g_L = 0.62425$, and $g_C = 0.9662$,

$$C_{\mathrm{HP}} = \frac{1}{2\pi \times 10^8 \times 0.62425}\ \mathrm{F} = 2.5495\ \mathrm{nF}$$

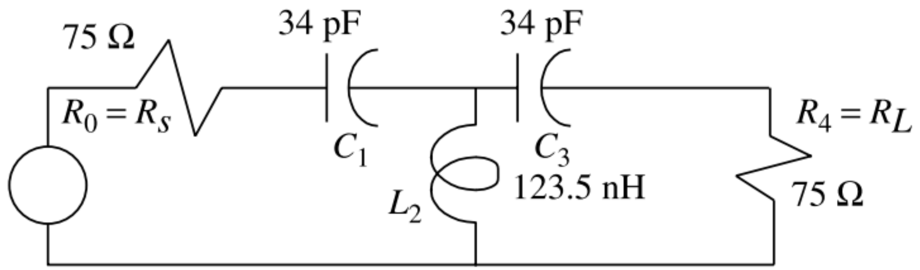
High Pass Filter

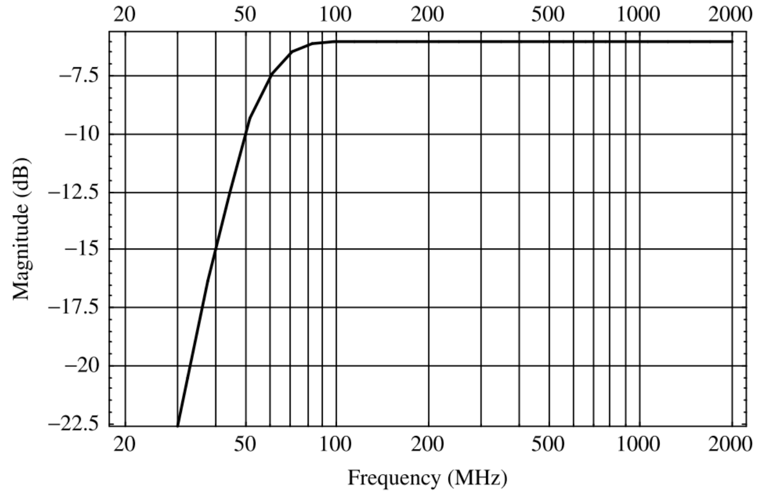
$$L_{HP} = \frac{1}{2\pi \times 10^8 \times 0.9662} \text{ H} = 1.6472 \text{ nH}$$

- applying the resistance scaling, we get

$$C_1 = C_3 = \frac{2.5495}{75} \text{ nF} \approx 34 \text{ pF}$$

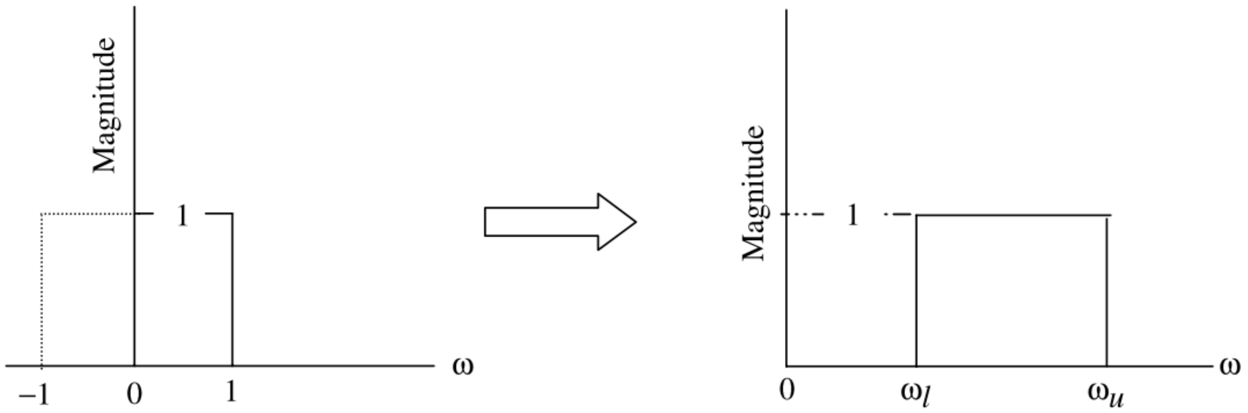
$$L_2 = 75 \times 1.6472 \text{ nH} = 123.5 \text{ nH}$$





Band Pass Filter

- A band-pass filter can be designed by
  - transforming the low-pass prototype
  - The frequency transformation is shown below



$$\omega_o = \sqrt{\omega_l \times \omega_u}$$

  - The frequency transformation is thus $\bar{\omega} = \dfrac{1}{\omega_u - \omega_l}\,\dfrac{\omega^2 - \omega_o^2}{\omega}$

Band Pass Filter

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- This transformation
  - replaces the series inductor of low-pass prototype with
    - an inductor L_{BP1} and a capacitor C_{BP1} that are connected in series.
  - The components values are determined as follows:

$$C_{BP1} = \frac{\omega_u - \omega_l}{\omega_o^2 g_L} \qquad L_{BP1} = \frac{g_L}{\omega_u - \omega_l}$$

- Also, capacitor C_{BP2} connected in parallel with inductor L_{BP2}
  - will replace the shunt capacitor of the low-pass prototype.

Band Pass Filter

- The components values are determined as follows:

$$L_{\mathrm{BP2}} = \frac{\omega_u - \omega_l}{\omega_o^2 g_C}$$

$$C_{\mathrm{BP2}} = \frac{g_C}{\omega_u - \omega_l}$$

- These elements need to be scaled further as desired by the load and source resistance.

Band Pass Filter

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- ***Example***
  - Design a bandpass Chebyshev filter that exhibits no more than 0.01-dB ripples in its passband. It must pass signals in the frequency band 10 to 40 MHz with zero insertion loss. Assume that the load and source resistances are at 75 Ω each.
- ***Solution***

$$f_o = \sqrt{f_l f_u} = \sqrt{10^7 \times 40 \times 10^6} = 20 \times 10^6 \text{ Hz}$$

  - With $m = 3$, $g_L = 0.62425$, and $g_C = 0.9662$,

$$C_{\text{BP1}} = \frac{2\pi \times 10^6 (40 - 10)}{\left(2\pi \times 20 \times 10^6\right)^2 \times 0.62425} \text{ F} = 19.922 \text{ nF}$$

Band Pass Filter

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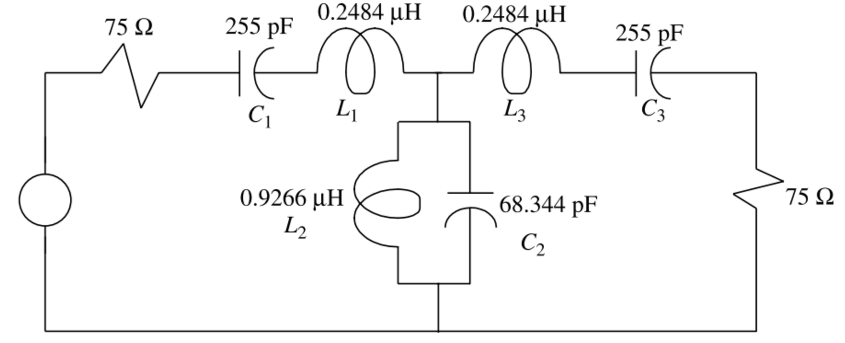
$$L_{BP1} = \frac{0.62424}{2\pi \times 30 \times 10^6} \text{H} = 3.3116\,\text{nH}$$

$$L_{BP2} = \frac{2\pi \times 10^6(40 - 10)}{(2\pi \times 20 \times 10^6)^2 \times 0.9662} \text{H} = 12.354\,\text{nH}$$

$$C_{BP2} = \frac{0.9662}{2\pi \times 30 \times 10^6} \text{F} = 5.1258\,\text{nF}$$



- applying the resistance scaling,

$$C_1 = C_3 = \frac{19.122}{75} \text{nF} = 254.96\,\text{pF} \approx 255\,\text{pF}$$

$$L_1 = L_3 = 75 \times 3.3116\,\text{nH} = 0.2484\,\mu\text{H}$$

$$L_2 = 75 \times 12.354\,\text{nH} = 0.9266\,\mu\text{H}$$

$$C_2 = \frac{5.1258}{75} \text{nF} = 68.344\,\text{pF}$$

Band Pass Filter

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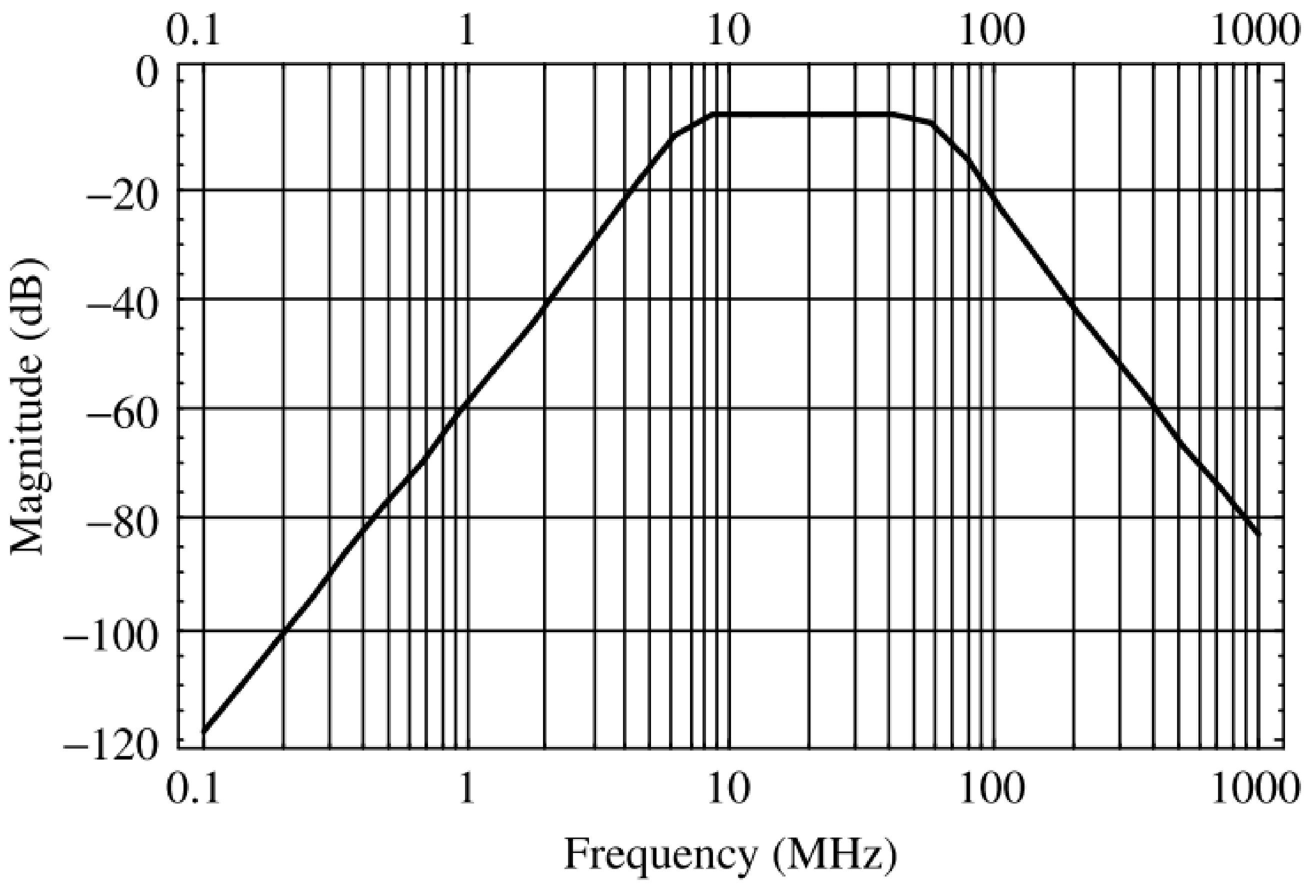

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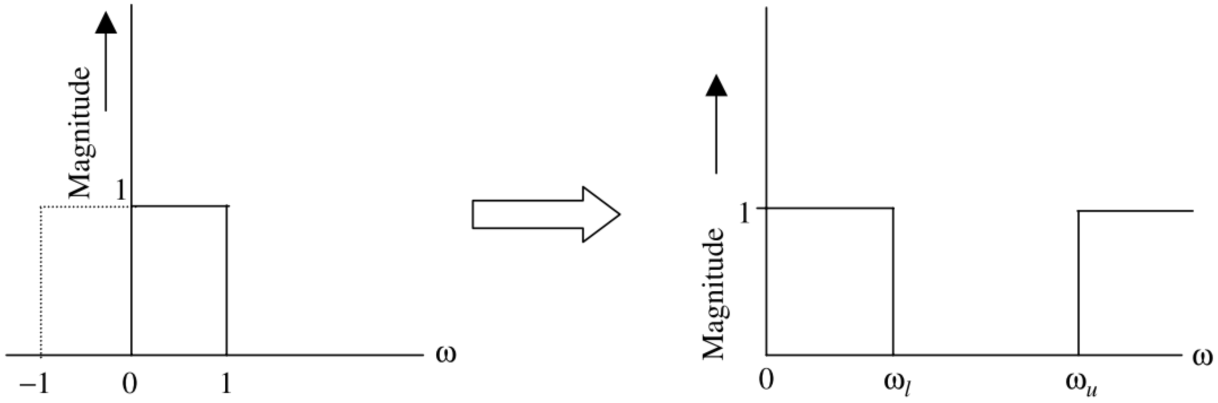
Band Stop Filter

- A band-pass filter can be realized by
  - transforming the low-pass prototype
  - The frequency transformation is shown below



$$\omega_o = \sqrt{\omega_l \times \omega_u}$$

  - The frequency transformation is thus $\bar{\omega} = (\omega_u - \omega_l)\dfrac{\omega}{\omega^2 - \omega_o^2}$

Band Stop Filter

- This transformation
  - replaces the series inductor of low-pass prototype with
    - an inductor L_{BS1} and a capacitor C_{BS1} that are connected in parallel.
  - The components values are determined as follows:

$$L_{BS1} = \frac{(\omega_u - \omega_l) g_L}{\omega_o^2}$$

$$C_{BS1} = \frac{1}{(\omega_u - \omega_l) g_L}$$

- Also, C_{BS2} which is connected in series with an inductor L_{BS2},
  - will replace the shunt capacitor of the low-pass prototype.

Band Stop Filter

- The components values are determined as follows:

$$L_{\mathrm{BS2}} = \frac{1}{(\omega_u - \omega_l)\, g_C}$$

$$C_{\mathrm{BS2}} = \frac{(\omega_u - \omega_l)\, g_C}{\omega_o^2}$$

- These elements need to be scaled further as desired by the load and source resistance.

Band Stop Filter

❖ *Example*

- Design a maximally flat bandstop filter with n = 3. It must stop signals in the frequency range 10 to 40 MHz and pass the rest of the frequencies. Assume that the load and source resistances are at 75 Ω each.

❖ *Solution*

$$f_o = \sqrt{f_l f_u} = \sqrt{10^7 \times 40 \times 10^6} = 20 \times 10^6 \text{ Hz}$$

- With $n = 3$, $g_L = 1$, and $g_C = 2$, from previous example,

$$L_{\text{BS1}} = \frac{2\pi \times 10^6 (40 - 10)}{(2\pi \times 20 \times 10^6)^2} \times 1 \text{ H} = 11.94 \text{ nH}$$

Band Stop Filter

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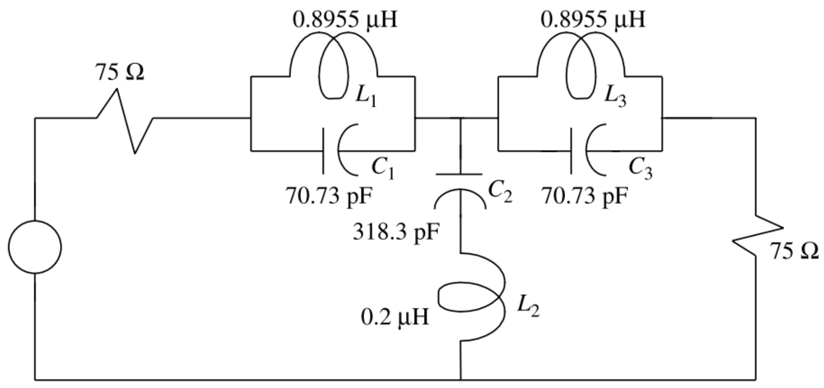
$$C_{BS1} = \frac{1}{2\pi \times 10^6 (40-10) \times 1} \text{ F} = 5.305 \text{ nF}$$

$$L_{BS2} = \frac{1}{2\pi \times 10^6 (40-10) \times 2} \text{H} = 2.653 \text{ nH}$$

$$C_{BS2} = \frac{2\pi \times 10^6 (40-10)}{(2\pi \times 20 \times 10^6)^2} \times 2 \text{ F} = 23.87 \text{ nF}$$

- applying the resistance scaling,



$$C_1 = C_3 = \frac{5.305}{75} \text{nF} = 70.73 \text{ pF}$$

$$L_1 = L_3 = 75 \times 11.94 \text{ nH} = 0.8955 \text{ μH}$$

$$L_2 = 75 \times 2.653 \text{ nH} = 0.1989 \text{ μH} \approx 0.2 \text{ μH}$$

$$C_2 = \frac{23.87}{75} \text{nF} = 318.3 \text{ pF}$$

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Band Stop Filter

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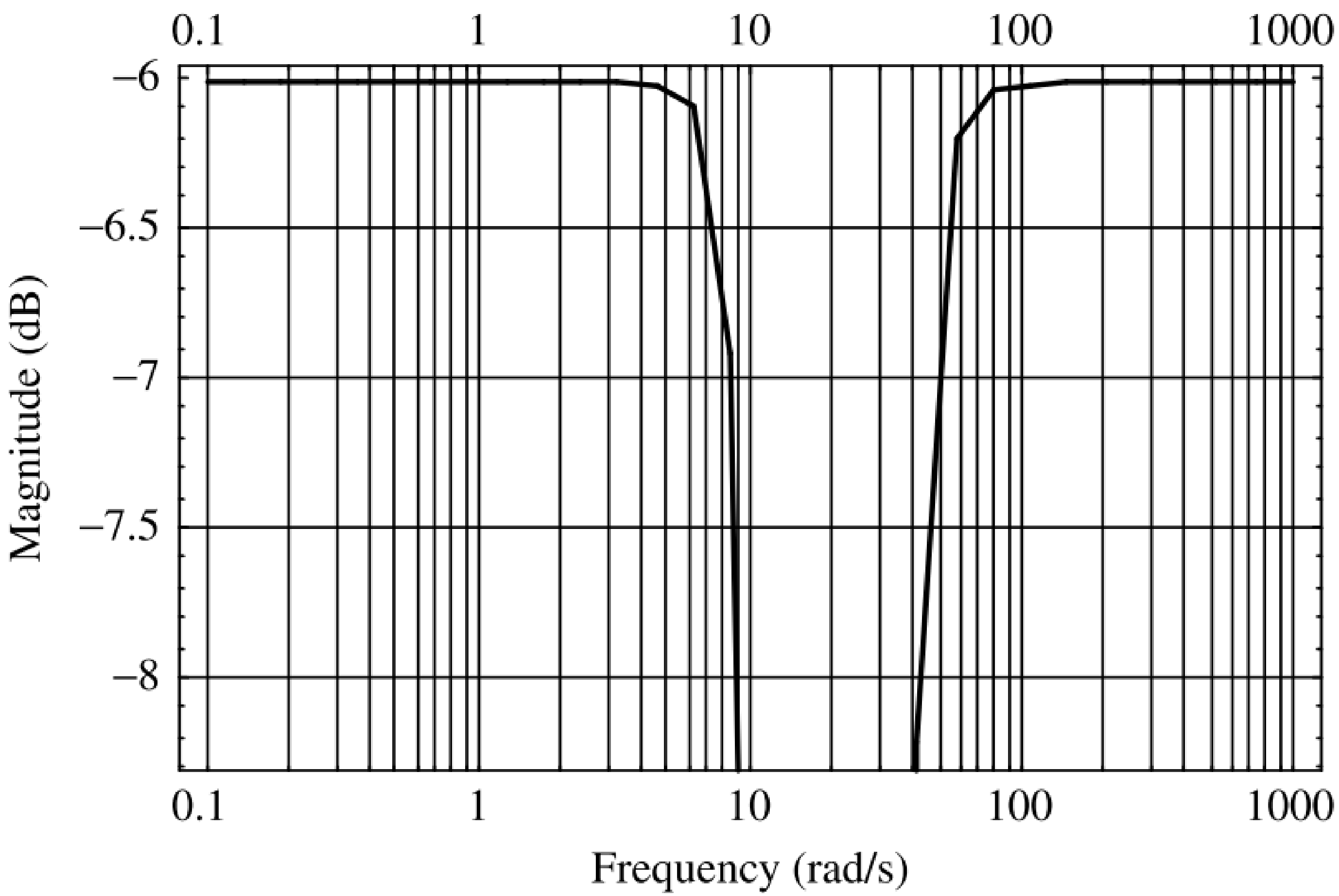

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