KNUST
Telecomm.
Engineering

TE 474

LECTURE 2:

Transmission Line Theory

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ICnSYS LAB

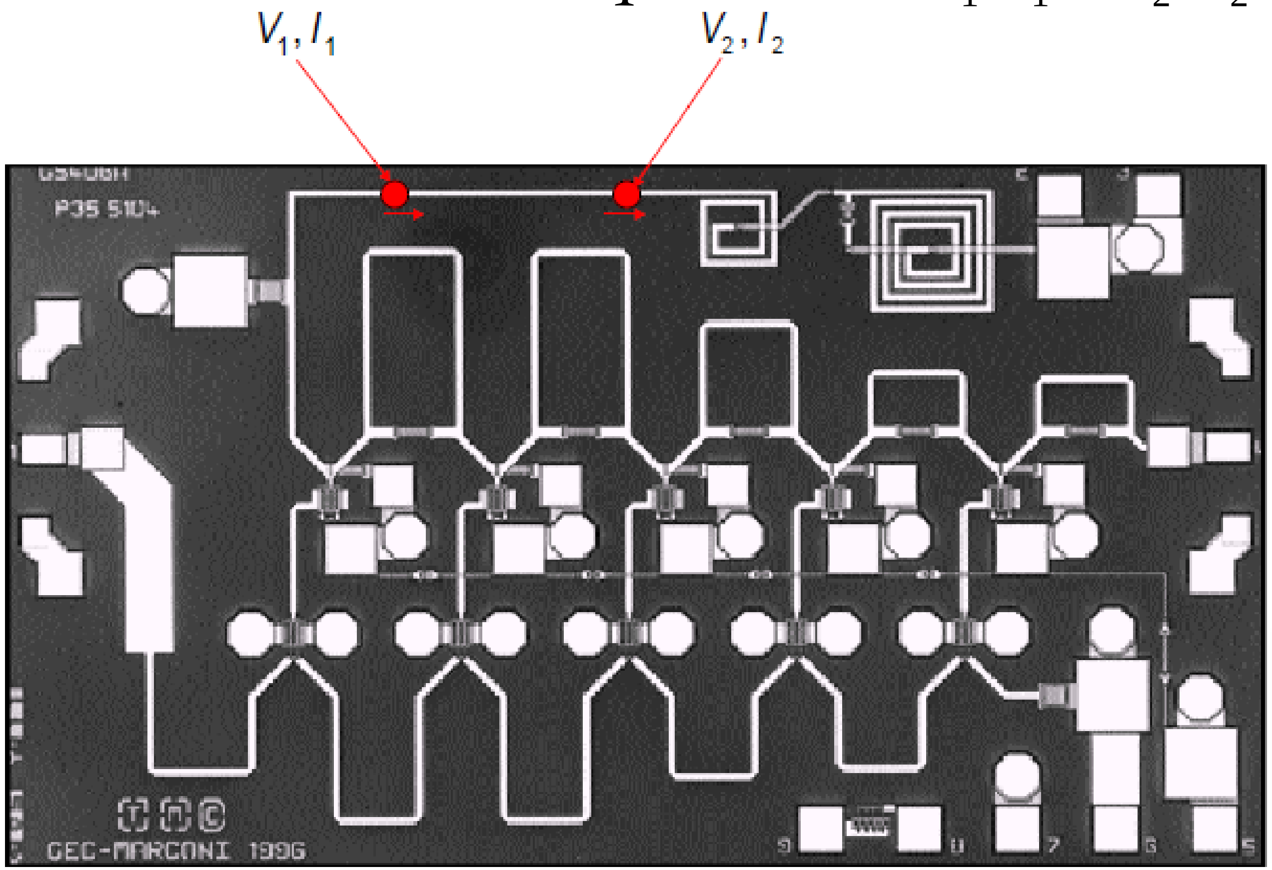
Presentation Layout

- Transmission Line (TL) Equation
- Parameters of TL
- Reflection and Return Loss
- Smith Chart
- Coaxial Lines

Transmission Line Equation

Introduction

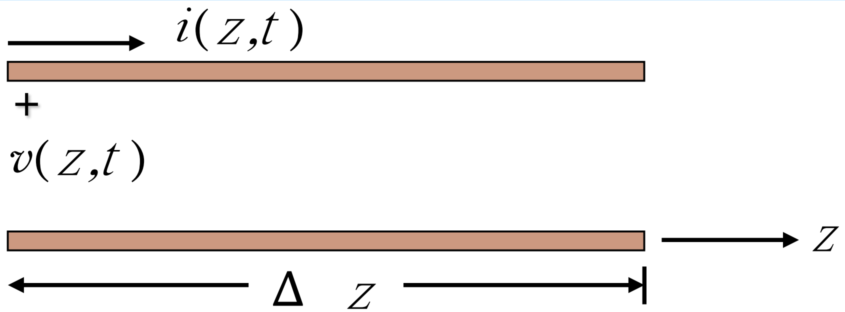
- At microwave frequencies, $V_1, I_1 \neq V_2, I_2$ respectively



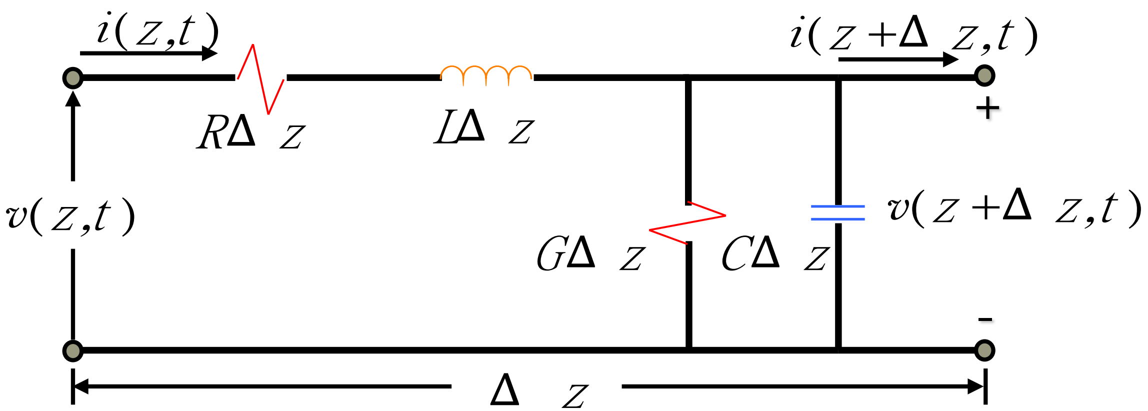
Assumption

Cross section is Uniform across propagating direction

Transmission Line Parameters



(a) Voltage and current definitions



(b) Lumped-element equivalent circuit

Fig. 2.1 Incremental length of transmission line

Transmission Line Parameters

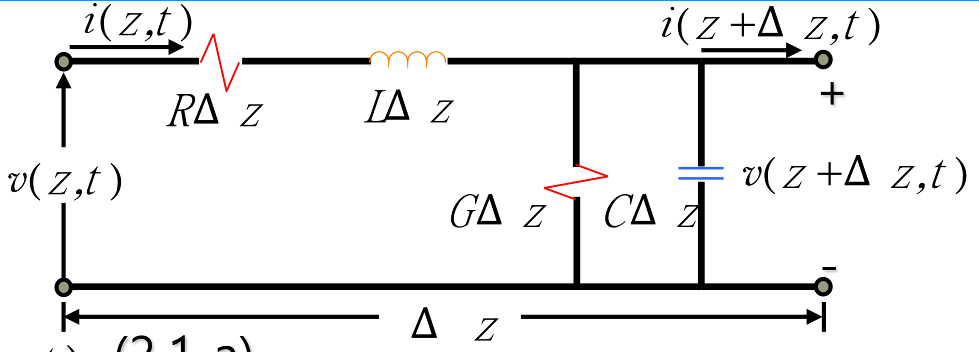
R : series resistance per unit length, for both conductors (conductor finite σ) (Ω/m)

L : series inductance per unit length, for both conductors (conductor wire's self inductance) (H/m)

G : shunt conductance per unit length (dielectric loss) (S/m)

C : shunt capacitance per unit length (capacitance between the conductors) (F/m)

Transmission Line Parameters



KVL : $v(z,t) - R\Delta z\, i(z,t) - L\Delta z \dfrac{\partial i(z,t)}{\partial t} = v(z+\Delta z,t)$ (2.1 a)

(2.1a) ➡ $\dfrac{v(z+\Delta z,t) - v(z,t)}{\Delta z} = -R\, i(z,t) - L\dfrac{\partial i(z,t)}{\partial t}$

$\Delta z \longrightarrow 0$ $\qquad \dfrac{\partial v(z,t)}{\partial z} = -R\, i(z,t) - L\dfrac{\partial i(z,t)}{\partial t}$

Similarly

$$\frac{\partial i(z,t)}{\partial z} = -G\, v(z,t) - C\frac{\partial v(z,t)}{\partial t}$$

Telegrapher equations

Transmission Line Parameters

Wave Propagation on a Transmission Line

$$\frac{d^2V(z)}{dz^2} - \gamma^2 V(z) = 0 \qquad (2.4\ a)$$

$$\frac{d^2I(z)}{dz^2} - \gamma^2 I(z) = 0 \qquad (2.4\ b)$$

wave equations

where $\gamma = \alpha + j\beta = \sqrt{(R + j\omega L)(G + j\omega C)}$

Solution of (2.4) is

$$V(z) = V_0^+ e^{-\gamma z} + V_0^- e^{\gamma z} \qquad (2.6\ a)$$

$$I(z) = I_0^+ e^{-\gamma z} + I_0^- e^{\gamma z} \qquad (2.6\ b)$$

Transmission Line Parameters

(2.6 a) into (2.3 a)

$$\left(※\ \frac{dV(z)}{dz} = -(R + j\omega L)I(z) \qquad \text{(2.3 a)} \right)$$

$$-\gamma V_0^+ e^{-\gamma z} + \gamma V_0^- e^{\gamma z} = -(R + j\omega L)I(z)$$

$$\therefore\ I(z) = \frac{\gamma}{R + j\omega L}(V_0^+ e^{-\gamma z} - V_0^- e^{\gamma z})$$

$$= \frac{1}{Z_0}(V_0^+ e^{-\gamma z} - V_0^- e^{\gamma z}) \qquad \text{(2.8)}$$

Z_0 : characteristic impedance

$$= \frac{R + j\omega L}{\gamma} = \frac{R + j\omega L}{\sqrt{(R + j\omega L)(G + j\omega C)}} = \sqrt{\frac{R + j\omega L}{G + j\omega C}}$$

Transmission Line Parameters

Lossless Line

$$\gamma = \alpha + j\beta = j\beta = j\omega\sqrt{LC}$$

$$(R = G = 0)$$

$$Z_0 = \sqrt{\frac{L}{C}}$$

$$\lambda = \frac{2\pi}{\beta} = \frac{2\pi}{\omega\sqrt{LC}} \quad \left(= \frac{V_p}{f} \right)$$

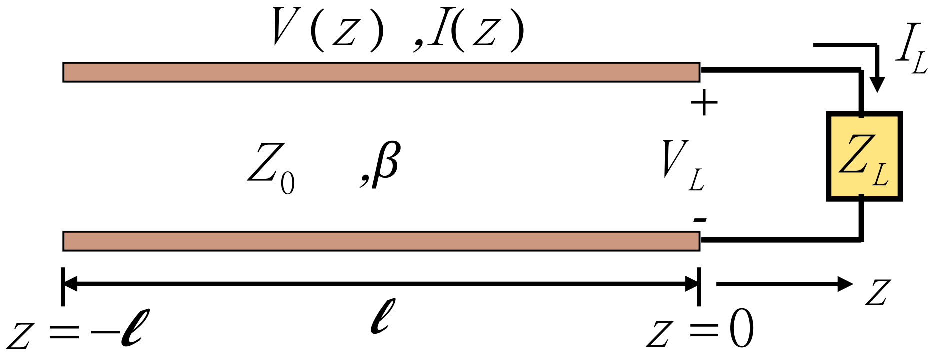
$$V_p = \frac{\omega}{\beta} = \frac{1}{\sqrt{LC}}$$

$$V(z) = V_0^+ e^{-j\beta z} + V_0^- e^{j\beta z} \qquad (2.14\text{ a})$$

$$I(z) = \frac{V_0^+}{Z_0} e^{-j\beta z} - \frac{V_0^-}{Z_0} e^{j\beta z} \qquad (2.14\text{ b})$$

Terminated Lossless Transmission Line

Transmission Line Parameters



$$V(z) = V_0^+ e^{-j\beta z} + V_0^- e^{j\beta z}$$

$$I(z) = \frac{1}{Z_0} [V_0^+ e^{-j\beta z} - V_0^- e^{j\beta z}]$$

At $z = 0$,

$$Z_L = \frac{V(0)}{I(0)} = \frac{V_L}{I_L} = Z_0 \frac{V_0^+ + V_0^-}{V_0^+ - V_0^-}$$

Transmission Line Parameters

$$\therefore V_0^- = \frac{Z_L - Z_0}{Z_L + Z_0} V_0^+ = \Gamma V_0^+$$

where $\Gamma = \dfrac{V_0^-}{V_0^+} = \dfrac{Z_L - Z_0}{Z_L + Z_0}$: voltage reflection coefficient

[Similar to for plane wave $\Gamma = \dfrac{\eta_L - \eta_0}{\eta_L + \eta_0}$]

$\Gamma = 0$ (No reflection)

➡

$$\therefore V(z) = V_0^+ (e^{-j\beta z} + \Gamma e^{j\beta z})$$

$$I(z) = \frac{V_0^+}{Z_0} (e^{-j\beta z} - \Gamma e^{j\beta z})$$

Transmission Line Parameters

Time-average power flow along the line

$$P_{av} = \frac{1}{2}\mathrm{Re}\left[V(z)I^*(z)\right]$$

$$= \frac{1}{2}\frac{\left|V_0^+\right|^2}{Z_0}\mathrm{Re}\left\{1 - \underbrace{\Gamma^* e^{-2j\beta z} + \Gamma e^{2j\beta z}}_{\text{purely imaginary}} - \left|\Gamma\right|^2\right\}$$

$$= \frac{1}{2}\frac{\left|V_0^+\right|^2}{Z_0}\left(1 - \left|\Gamma\right|^2\right)$$

Return Loss (RL)

$$RL = -20\log\left|\Gamma\right| \ (\mathrm{dB})$$

$\Gamma = 0$; match — $RL = \infty$ dB

$\left|\Gamma\right| = 1$; open or short — $RL = 0$ dB

Transmission Line Parameters

V_{max}, V_{min}, SWR

$$V(z) = V_0^+ (e^{-j\beta z} + \Gamma e^{j\beta z}) = V_0^+ (e^{-j\beta z} + |\Gamma| e^{j(\beta z + \theta)})$$

$$(\because \Gamma = |\Gamma| e^{j\theta})$$

$$= V_0^+ \left[(\cos\beta z - j\sin\beta z) + |\Gamma| \{ \cos(\beta z + \theta) + j\sin(\beta z + \theta) \} \right]$$

$$= V_0^+ \left[(\cos\beta z + |\Gamma|\cos(\beta z + \theta)) + j\{ |\Gamma|\sin(\beta z + \theta) - \sin\beta z \} \right]$$

$$\therefore |V(z)|$$

$$= |V_0^+| \sqrt{ \{\cos\beta z + |\Gamma|\cos(\beta z + \theta)\}^2 + \{|\Gamma|\sin(\beta z + \theta) - \sin\beta z\}^2 }$$

$$= |V_0^+| \sqrt{ 1 + 2|\Gamma|\cos\beta z \cdot \cos(\beta z + \theta) - 2|\Gamma|\sin\beta z \cdot \sin(\beta z + \theta) + |\Gamma|^2 }$$

$$= |V_0^+| \sqrt{ 1 + 2|\Gamma|\cos(2\beta z + \theta) + |\Gamma|^2 }$$

Transmission Line Parameters

$$\therefore \; V_{max} = |V_0^+|(1+|\Gamma|)$$

$$(\cos(2\beta z + \theta) = 1)$$

→ distance between successive maxima is $\lambda/2$

$$V_{min} = |V_0^+|(1-|\Gamma|)$$

$$(\cos(2\beta z + \theta) = -1)$$

→ distance between successive minima is $\lambda/2$

(Voltage) Standing Wave Ratio : SWR or VSWR

$$SWR = \frac{V_{max}}{V_{min}} = \frac{1+|\Gamma|}{1-|\Gamma|}$$

$1 \le SWR < \infty$, $SWR = 1$ $|\Gamma| = 0$ and matched

Transmission Line Parameters

Reflection Coefficient at $z = -\ell$

$$V(z) = V_0^+ e^{-j\beta z} + V_0^- e^{j\beta z} \quad \text{at} \quad z = -\ell$$

$$\Gamma(-\ell) = \frac{V^-(-\ell)}{V^+(-\ell)} = \frac{V_0^- e^{-j\beta\ell}}{V_0^+ e^{j\beta\ell}} = \frac{V_0^-}{V_0^+} e^{-j2\beta\ell} = \Gamma(0) e^{-2j\beta\ell}$$

Input impedance at $z = -\ell$

$$Z_{in} = \frac{V(-\ell)}{I(-\ell)} = \frac{V_0^+ (e^{j\beta\ell} + \Gamma e^{-j\beta\ell})}{V_0^+ (e^{j\beta\ell} - \Gamma e^{-j\beta\ell})} Z_0 = \frac{1 + \Gamma e^{-j2\beta\ell}}{1 - \Gamma e^{-j2\beta\ell}} Z_0$$

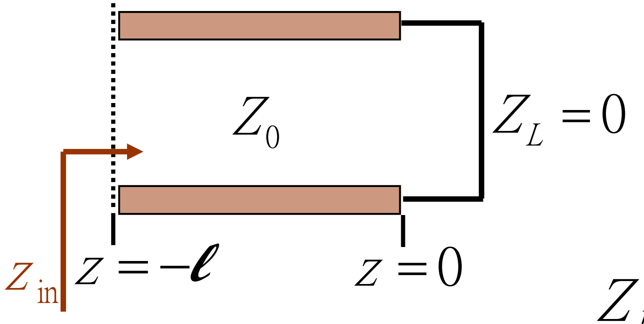
$$= Z_0 \frac{Z_L + jZ_0 \tan\beta\ell}{Z_0 + jZ_L \tan\beta\ell}$$

(refer to (2.44))

$$\left[\because \Gamma = \frac{Z_L - Z_0}{Z_L + Z_0} \right]$$

Special Cases of Lossless Terminated Lines

① Short-circuited Termination



$$Z_L = 0 \qquad \therefore \;\; \Gamma = \frac{Z_L - Z_0}{Z_L + Z_0} = -1$$

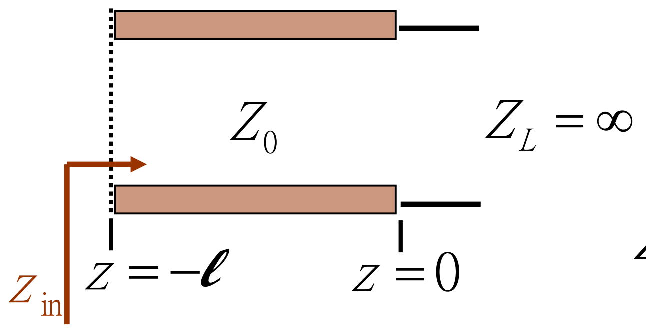
$$V(z) = V_0^+ (e^{-j\beta z} - e^{j\beta z}) = -2jV_0^+ \sin\beta z$$

$$I(z) = \frac{V_0^+}{Z_0}(e^{-j\beta z} + e^{j\beta z}) = \frac{2V_0^+}{Z_0}\cos\beta z$$

$$Z_{in} = \frac{V(-\ell)}{I(-\ell)} = jZ_0 \tan\beta\ell$$

Special Cases of Lossless Terminated Lines

② Open-circuited Termination



$$Z_L = \infty$$

$$\therefore \quad \Gamma = \frac{Z_L - Z_0}{Z_L + Z_0} = 1$$

$$V(z) = V_0^+ (e^{-j\beta z} + e^{j\beta z}) = 2V_0^+ \cos \beta z$$

$$I(z) = \frac{V_0^+}{Z_0} (e^{-j\beta z} - e^{j\beta z}) = \frac{-2jV_0^+}{Z_0} \sin \beta z$$

$$Z_{in} = -jZ_0 \cot \beta \ell$$

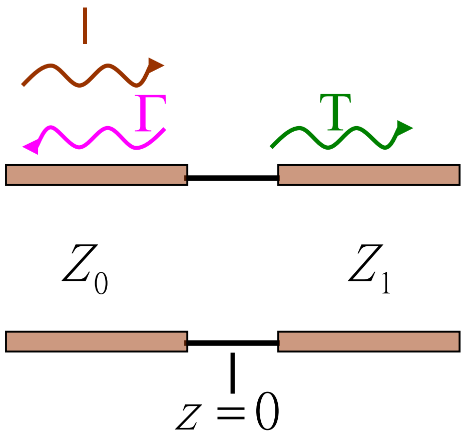
Special Cases of Lossless Terminated Lines

$$Z_{in} = Z_0 \frac{Z_L + jZ_0 \tan\beta \ell}{Z_0 + jZ_L \tan\beta \ell}$$

$$\ell = \frac{\lambda}{2} \qquad \beta \ell = \frac{2\pi}{\lambda} \times \frac{\lambda}{2} = \pi \qquad \therefore Z_{in} = Z_L$$

$$\ell = \frac{\lambda}{4} \qquad \beta \ell = \frac{\pi}{2} \qquad \therefore Z_{in} = \frac{Z_0^2}{Z_L}$$

→ quarter-wave transformer



$$\Gamma = \frac{Z_1 - Z_0}{Z_1 + Z_0}$$

$z < 0$ $\quad V(z) = V_0^+ (e^{-j\beta z} + \Gamma e^{j\beta z})$

$z > 0$ $\quad V(z) = V_0^+ \mathrm{T} e^{-j\beta z}$

$z = 0$ $\quad$ Since $V(0)$ must be same,

$$\mathrm{T} = 1 + \Gamma = 1 + \frac{Z_1 - Z_0}{Z_1 + Z_0} = \frac{2Z_1}{Z_1 + Z_0}$$

Smith Chart

- graphical aid useful for solving t-line problems.
- Developed by P. Smith in 1939 at Bell Telephone Laboratories
- Enables visualization of transmission line problems without requiring much calculation
- Smith chart to the M/W engineer is useful for developing intuition of transmission line problems.

Smith Chart

Example: Basic Smith Chart Operations

Load impedance : $130 + j90 \ \Omega$

X'tics impedance of t-line: $50 \ \Omega$

Length of line: 0.3λ

$\Gamma(0)$, $\Gamma(-0.3\lambda)$, Z_{in}, SWR on the line, Return loss?

Smith Chart-Transmission Line Problem

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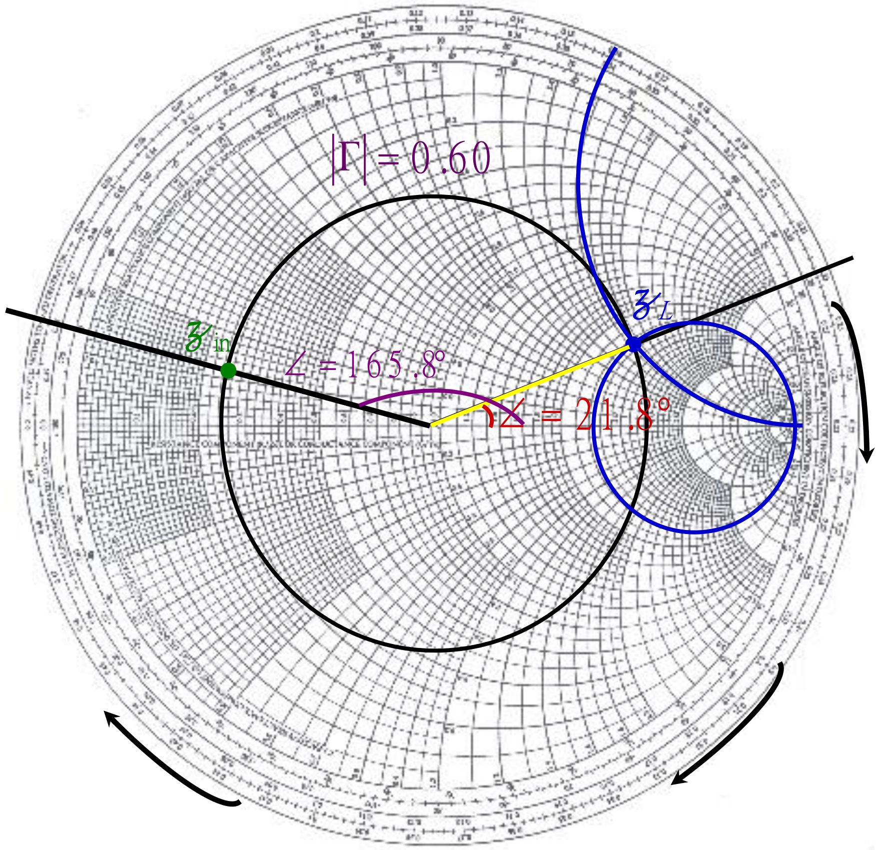

① $z_L = \frac{Z_L}{Z_0} = 2.60 + j1.80$ plotted.

② Using Compass or rule

$|\Gamma| = 0.6,\ \mathrm{SWR} = 3.98,$

$\mathrm{RL} = 4.4\mathrm{dB},\ \theta = 21.8°$

③ 0.220λ Direction wavelengths-toward-generator move

0.3λ to 0.520λ

With period of 0.5λ

Equivalent to 0.020λ

④ Reading

$z_{in} = 0.255 + j0.117$

$\therefore Z_{in} = z_{in} Z_0 = 12.7 + j5.8$

⑤ $|\Gamma(-0.3\lambda)| = |\Gamma(0)|$ $\theta(-0.3\lambda) = 165.8°$

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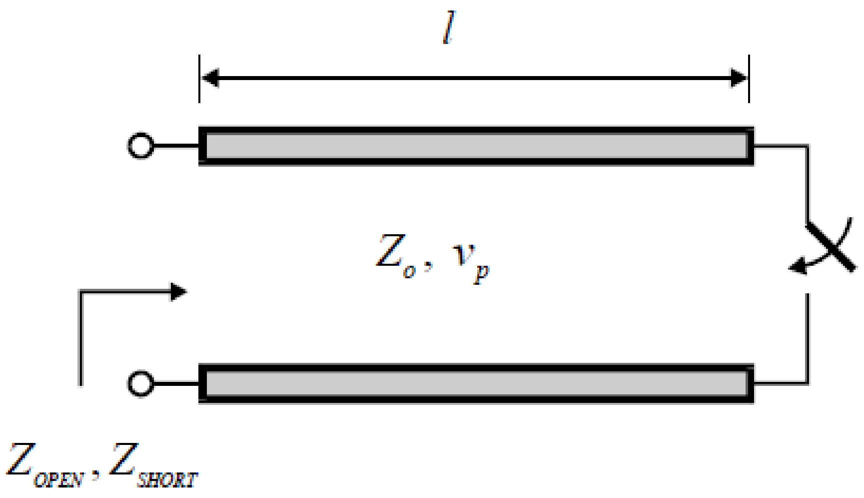
Measurement of Z_o

Impedance method(High Frequency)



End Terminal Open $Z_{OPEN} = \dfrac{1}{j\omega C_m l}$

End Terminal Short $Z_{SHORT} = j\omega L_m l$

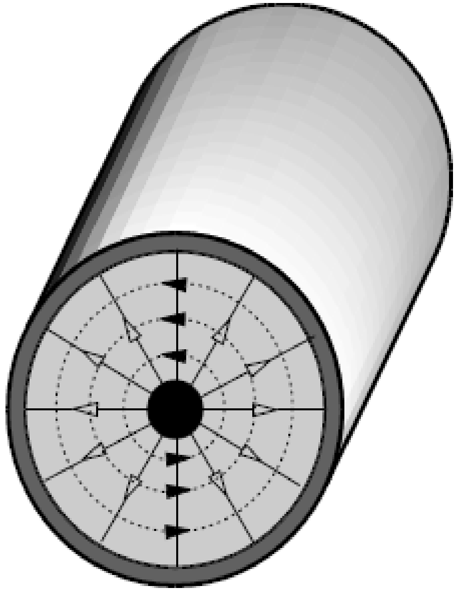
$$Z_o = \sqrt{\frac{L}{C}} = \sqrt{Z_{OPEN} \cdot Z_{SHORT}}$$

$$v_p = \frac{1}{\sqrt{LC}} = \frac{1}{\omega l}\sqrt{\left|\frac{Z_{OPEN}}{Z_{SHORT}}\right|}$$

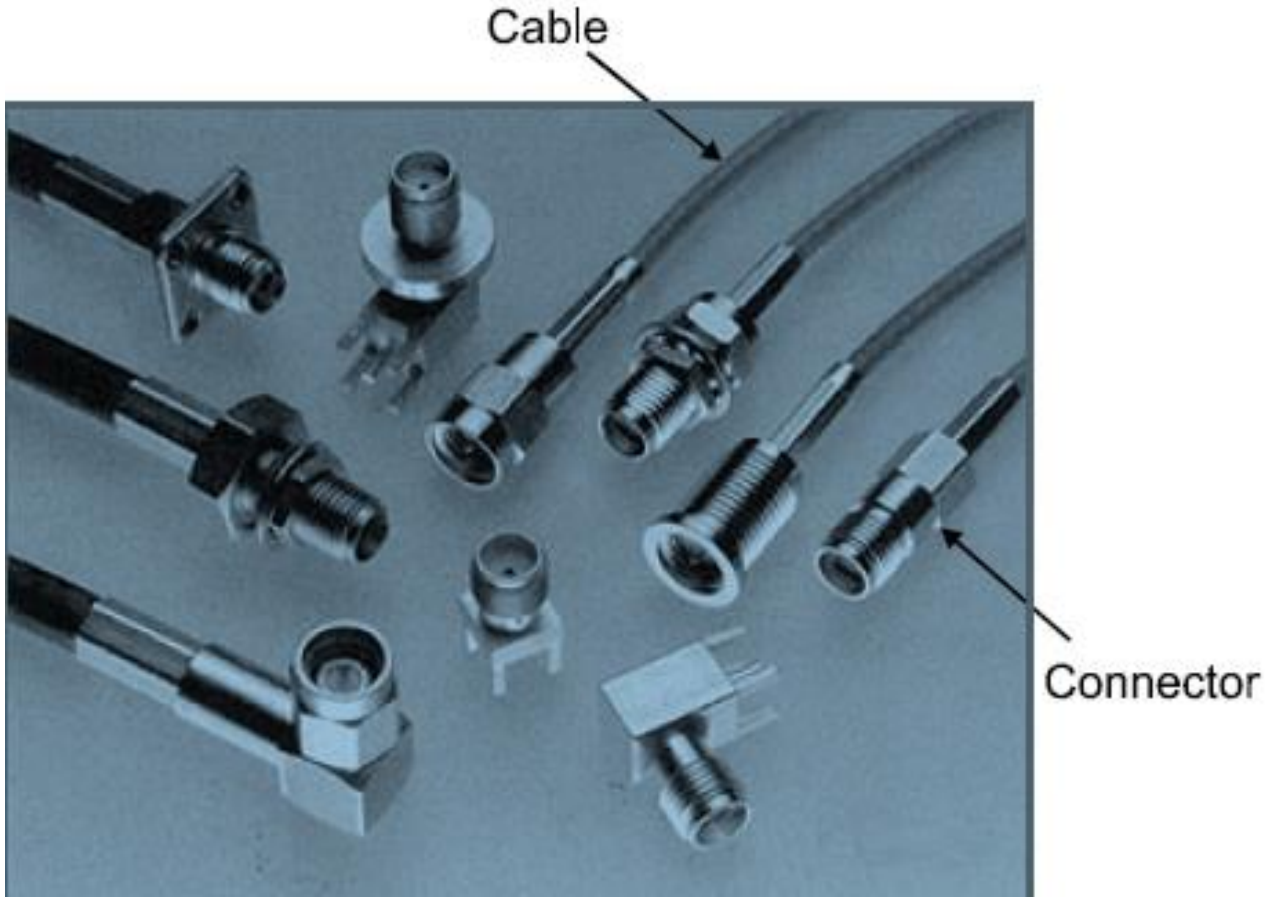
Coaxial Transmission Lines

Coaxial Lines

- Used for measurement
  - Construction: Connector, Cable and Adapter
- Cable:
  - Flexible, Semi-Rigid, Handy reformable
- Connectors:
  - BNC, TNC, SMA
  - Precision; 7.0mm, , 3.5mm, 2.9mm, 2.4mm, 1.0mm



Coaxial Lines



Photograph of Coaxial Transmission

Coaxial Line Connectors

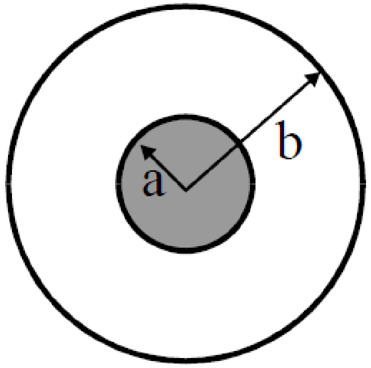
- SMA Adaptors

- BNC, TNC, SMA

Photograph of Coaxial Connectors

Simulation Project 1

- A kind of coaxial cable RG-142 has an inner radius a of 0.035 inch and an outer radius b, of 0.116 inch. By using ADS, follow the open -short procedure and express the characteristics impedance of the coaxial cable in terms of the ratio a/b, where a and b are the inner and outer radii respectively shown in Fig. 3E.3 below. Compare the results obtained with that obtained from the formula



$$Z_o = 60\sqrt{\frac{\mu_r}{\varepsilon_r}}\ln\left(\frac{a}{b}\right)$$