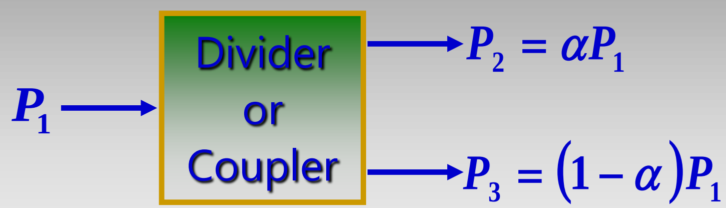
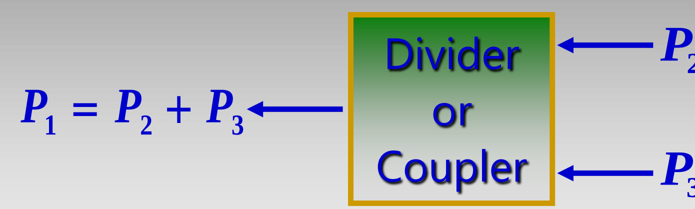
Ch. 7 Power Dividers and Directional Couplers

Power divider or combiner

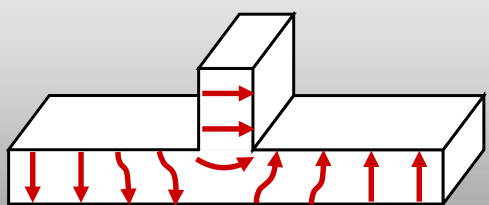


<Power division>

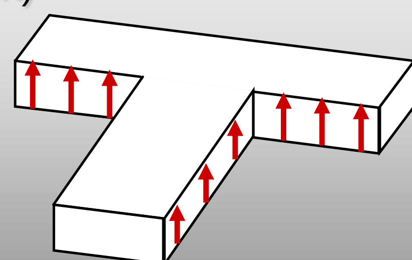


<Power combining>

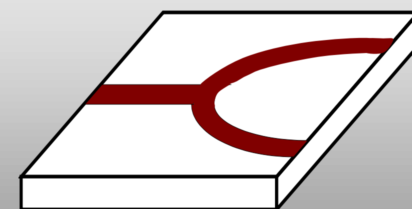
- T-junctions (3-port network)



< E-plane waveguide T>



<H-plane waveguide T>



<Microstrip T-junction>

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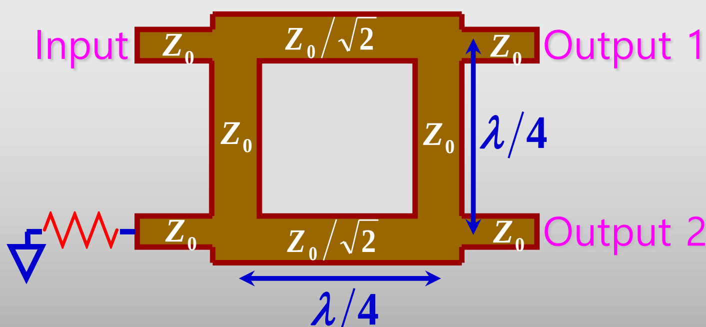
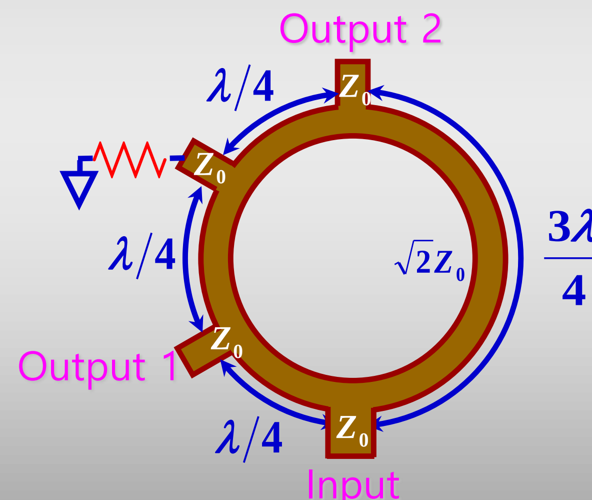
- Directional couplers (4-port network)
  - arbitrary power division
    (10 dB, 20 dB, 3 dB, etc.)

- Hybrid junctions (4-port network)
  - equal power division (3 dB)



< $90°$ phase difference>
(quadrature hybrid)



<$180°$ phase difference>
(ring hybrid)

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HISTORY

1940

- US MIT Radiation Laboratory developed mainly waveguide-type coupler, power divider, T junction etc.

Mid 1950~ 1960

- development of devices for mainly planar stripline and also microstrip circuits. (Wilkinson divider, branch line hybrid, coupled line directional coupler etc)

7.1 Basic Properties of Dividers and Couplers

Three-Port Networks (T-junctions)

$[S]$ of an arbitrary 3-port network

$$[S]=\begin{bmatrix} S_{11} & S_{12} & S_{13} \\ S_{21} & S_{22} & S_{23} \\ S_{31} & S_{32} & S_{33} \end{bmatrix}$$ (9 independent elements)

If the component is passive and contains no anisotropic materials,

① network is reciprocal and $S_{ij} = S_{ji}$ ($[S]$ is symmetric) (Recall)

If the component is lossless and all ports are matched,

② $[S^*]^t[S]=[U]$ ($[S]$ is a unitary matrix) (Recall)

③ $S_{ii} = 0$

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From conditions ①, ③

$$[S]=\begin{bmatrix} S_{11} & S_{12} & S_{13} \\ S_{21} & S_{22} & S_{23} \\ S_{31} & S_{32} & S_{33} \end{bmatrix} \Rightarrow [S]=\begin{bmatrix} 0 & S_{12} & S_{13} \\ S_{12} & 0 & S_{23} \\ S_{13} & S_{23} & 0 \end{bmatrix}$$

From conditions ②

$$[S^*]^t[S]=[U] \Rightarrow \begin{bmatrix} 0 & S_{12}^* & S_{13}^* \\ S_{12}^* & 0 & S_{23}^* \\ S_{13}^* & S_{23}^* & 0 \end{bmatrix}\begin{bmatrix} 0 & S_{12} & S_{13} \\ S_{12} & 0 & S_{23} \\ S_{13} & S_{23} & 0 \end{bmatrix}=\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$|S_{12}|^2+|S_{13}|^2=1,\ |S_{12}|^2+|S_{23}|^2=1,\ |S_{13}|^2+|S_{23}|^2=1 \quad (7.3\ a\sim c)$$

$$S_{13}^*S_{23}=0\ (r\ 1\ c\ 2),\ S_{23}^*S_{12}=0\ (r\ 3\ c\ 1),\ S_{12}^*S_{13}=0\ (r\ 2\ c\ 3) \quad (7.3\ d\sim f)$$

from (7.3 d~f) S_{13}, S_{23}, S_{12} 2 out of the 3 parameters must be 0.

In that case however (7.3 a~c) will not be satisfied.

∴ A Lossless, reciprocal 3-port network cannot be matched at all ports:
- 3 port, Lossless, reciprocal, all ports matched: Not physically realizable

However, relaxing one of the 3 conditions makes it physically realizable.

Circulator

A key physically realizable 3-port network.

(matched, lossless, but nonreciprocal)

$$[S]=\begin{bmatrix} 0 & S_{12} & S_{13} \\ S_{21} & 0 & S_{23} \\ S_{31} & S_{32} & 0 \end{bmatrix}$$

$$[S^*]^t[S]=\begin{bmatrix} 0 & S_{21}^* & S_{31}^* \\ S_{12}^* & 0 & S_{32}^* \\ S_{13}^* & S_{23}^* & 0 \end{bmatrix}\begin{bmatrix} 0 & S_{12} & S_{13} \\ S_{21} & 0 & S_{23} \\ S_{31} & S_{32} & 0 \end{bmatrix}=\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}=[U]$$ from this,

$S_{31}^*S_{32}=0$ (r 1 c 2) , $S_{21}^*S_{23}=0$ (r 1 c 3) , $S_{12}^*S_{13}=0$ (r 2 c 3)

(7.5 a~c)

$|S_{21}|^2+|S_{31}|^2=1$, $|S_{12}|^2+|S_{32}|^2=1$, $|S_{13}|^2+|S_{23}|^2=1$ (7.5 d~f)

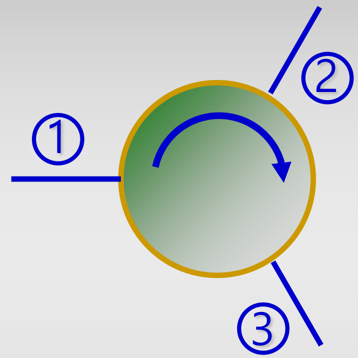
For equations (7.5 a~f) to be satisified,

$$S_{12} = S_{23} = S_{31} = 0 \quad , \quad |S_{21}| = |S_{32}| = |S_{13}| = 1 \tag{7.6 a}$$

or

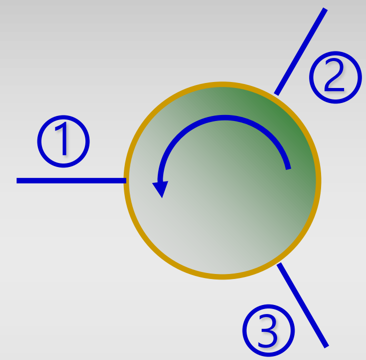
$$S_{21} = S_{32} = S_{13} = 0 \quad , \quad |S_{12}| = |S_{23}| = |S_{31}| = 1 \tag{7.6 b}$$

$$[S] = \begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$$



<Clockwise circulation>

$$[S] = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix}$$



<Counterclockwise circulation>

For this example of Physically realizable 3-port network

- If Lossless, reciprocal only 2 ports matched
- If Matched, reciprocal , then lossy circuit: resistive divider

Four-Port Networks (Directional Couplers)

If the 4-port network is matched, reciprocal, and lossless

$$[S^*]^t[S]=[U] \quad \text{must be satisfied}$$

$$\begin{bmatrix} 0 & S_{12}^* & S_{13}^* & S_{14}^* \\ S_{12}^* & 0 & S_{23}^* & S_{24}^* \\ S_{13}^* & S_{23}^* & 0 & S_{34}^* \\ S_{14}^* & S_{24}^* & S_{34}^* & 0 \end{bmatrix} \begin{bmatrix} 0 & S_{12} & S_{13} & S_{14} \\ S_{12} & 0 & S_{23} & S_{24} \\ S_{13} & S_{23} & 0 & S_{34} \\ S_{14} & S_{24} & S_{34} & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$S_{13}^* S_{23} + S_{14}^* S_{24} = 0$ (r 1 c 2) , $S_{14}^* S_{13} + S_{24}^* S_{23} = 0$ (r 4 c 3) (7.10 a~b)

Multiply (7.10 a) by S_{24}^* and (7.10 b) by S_{13}^* and subtract the results

$$S_{14}^*\left(|S_{13}|^2 - |S_{24}|^2\right) = 0 \qquad (7.11)$$

Similarly

$S_{12}^* S_{23} + S_{14}^* S_{34} = 0$ (r 1 c 3) , $S_{14}^* S_{12} + S_{34}^* S_{23} = 0$ (r 4 c 2) (7.12 a~b)

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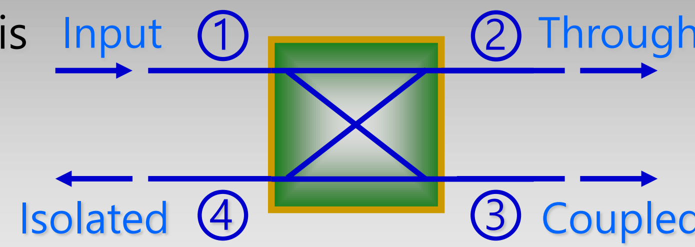
Multiply (7.12 a) by S_{12} and (7.12 b) by S_{34} and subtract the results

$$S_{23}\left(|S_{12}|^2 - |S_{34}|^2\right) = 0 \qquad (7.13)$$

The only way to satisfy (7.11) and (7.13) is



$$S_{14} = S_{23} = 0$$

(x,tics of directional coupler)

With this conditions, equations for the principal diagonal are:

$$|S_{12}|^2 + |S_{13}|^2 = 1\ ,\ |S_{12}|^2 + |S_{24}|^2 = 1\ ,\ |S_{13}|^2 + |S_{34}|^2 = 1\ ,\ |S_{24}|^2 + |S_{34}|^2 = 1 \qquad (7.14\ a\sim d)$$

Thus, from (7.14 a, b) , $|S_{13}| = |S_{24}|$ and

from (7.14 b, d) $|S_{12}| = |S_{34}|$

For simplicity, set. $S_{12} = S_{34} = \alpha$, $S_{13} = \beta e^{j\theta}$, $S_{24} = \beta e^{j\phi}$

Multiplying row2 by column 3 $S_{12}^{*}S_{13} + S_{24}^{*}S_{34} = 0$ and substituting above,

$$\alpha\beta e^{j\theta} + \alpha\beta e^{-j\phi} = 0 \quad \Rightarrow \quad \therefore\ e^{j\theta} + e^{-j\phi} = 0$$

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Two practically useful coupler $\theta = \phi = \pi/2$ and $\theta = 0\,,\ \phi = \pi$

① Symmetrical coupler

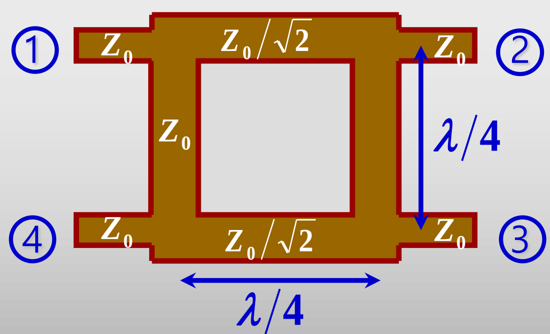
$(\theta = \phi = \pi/2)$

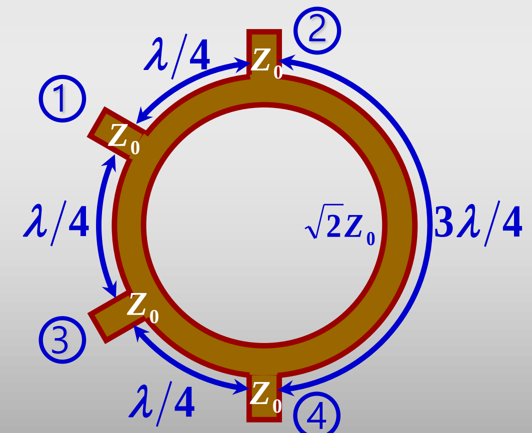
$$[s] = \begin{bmatrix} 0 & \alpha & \beta & 0 \\ \alpha & 0 & 0 & -\beta \\ \beta & 0 & 0 & \alpha \\ 0 & -\beta & \alpha & 0 \end{bmatrix}$$

② Assymmetrical coupler

$(\theta = 0\,,\ \phi = \pi)$

$$[s] = \begin{bmatrix} 0 & \alpha & j\beta & 0 \\ \alpha & 0 & 0 & j\beta \\ j\beta & 0 & 0 & \alpha \\ 0 & j\beta & \alpha & 0 \end{bmatrix}$$

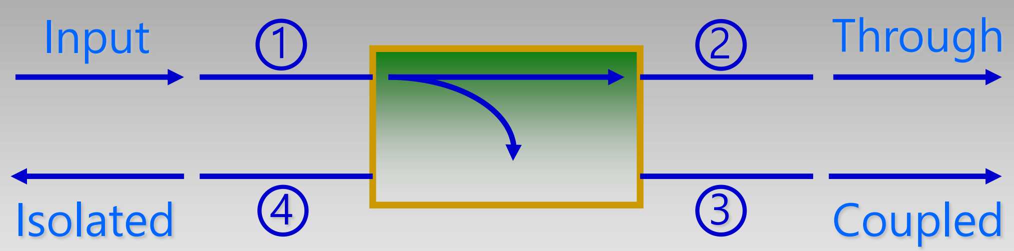


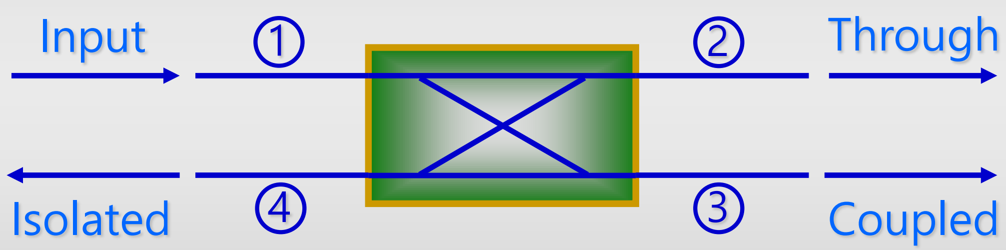


In both cases. $\alpha^2 + \beta^2 = 1$

In conclusion, a matched, reciprocal, lossless 4-port network is a directional coupler.

Directional coupler





Port 1 : Power input port

Port 3 : coupled port with coupling factor $|S_{13}|^2 = \beta^2$

Port 2 : remaining power is $|S_{12}|^2 = \alpha^2 = 1 - \beta^2$ (through port)

Port 4 : Ideally , no power is transmitted (isolated port)

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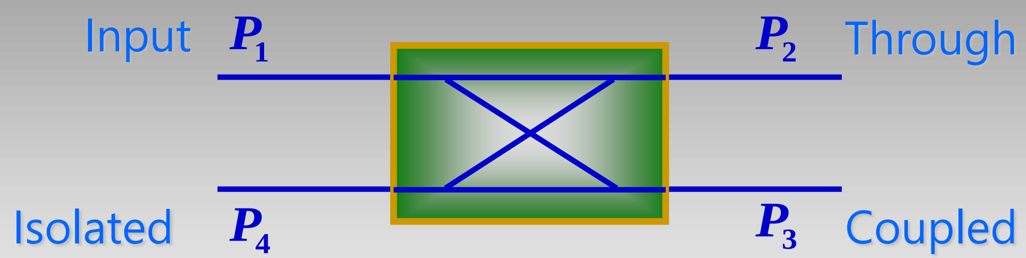
Definition of Terms relevant to Directional coupler



Coupling $= C = 10\log\dfrac{P_1}{P_3} = 10\log\dfrac{1}{\beta^2} = -20\log\beta$ (dB)

Directivity $= D = 10\log\dfrac{P_3}{P_4} = 10\log\dfrac{P_3}{P_1}\dfrac{P_1}{P_4} = 10\log\dfrac{\beta^2}{|S_{14}|^2}$

$= 20\log\dfrac{\beta}{|S_{14}|}$ (dB)

Isolation $= I = 10\log\dfrac{P_1}{P_4} = -20\log|S_{14}|$ (dB)

($I = D + C$ (dB))

Eg.①) For a Coupling factor of 3 dB $\left(\alpha = \beta = 1/\sqrt{2}\right)$ and a phase Difference of $90°$ between port 2 and port 3, we have

➡ quadrature hybrid

$$[s] = \begin{bmatrix} 0 & \alpha & j\beta & 0 \\ \alpha & 0 & 0 & j\beta \\ j\beta & 0 & 0 & \alpha \\ 0 & j\beta & \alpha & 0 \end{bmatrix} \Rightarrow [s] = \frac{1}{\sqrt{2}} \begin{bmatrix} 0 & 1 & j & 0 \\ 1 & 0 & 0 & j \\ j & 0 & 0 & 1 \\ 0 & j & 1 & 0 \end{bmatrix}$$

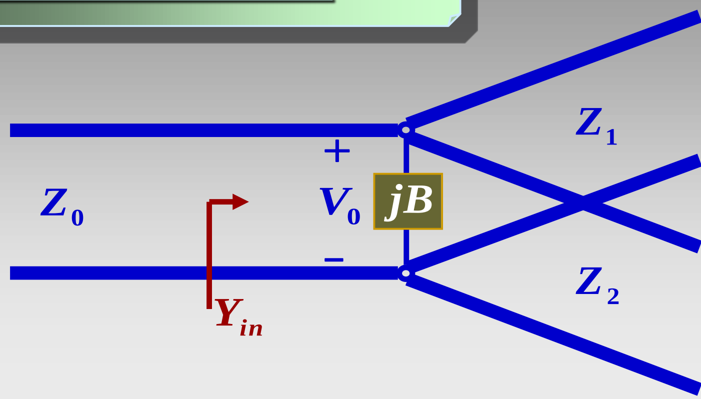
Eg.②) For a Coupling factor of 3 dB $\left(\alpha = \beta = 1/\sqrt{2}\right)$ and phase difference of $180°$ between ports 4 and 1, we have

➡ magic-T or rat-race hybrid

$$[s] = \begin{bmatrix} 0 & \alpha & \beta & 0 \\ \alpha & 0 & 0 & -\beta \\ \beta & 0 & 0 & \alpha \\ 0 & -\beta & \alpha & 0 \end{bmatrix} \Rightarrow [s] = \frac{1}{\sqrt{2}} \begin{bmatrix} 0 & 1 & 1 & 0 \\ 1 & 0 & 0 & -1 \\ 1 & 0 & 0 & 1 \\ 0 & -1 & 1 & 0 \end{bmatrix}$$

7.2 T-junction Power Divider

Lossless Divider



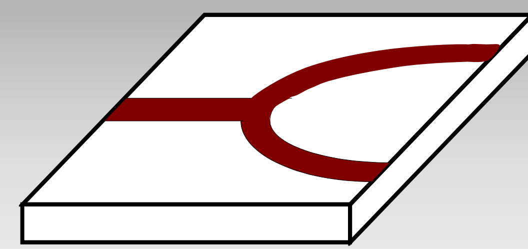


Fig. 7.6 Transmission line model of a lossless T-junction

Match condition

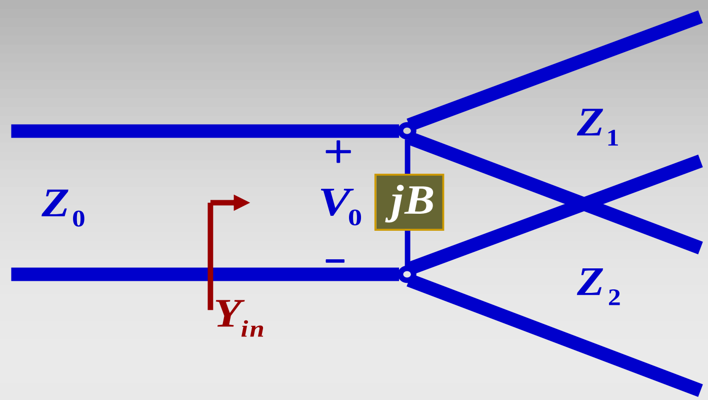
$$Y_{in} = jB + \frac{1}{Z_1} + \frac{1}{Z_2} = \frac{1}{Z_0}$$

$\boldsymbol{B}$: stored energy at the junction discontinuity

Z_1 , Z_2 Enable the setting of power distribution ratio

Eg. 7.1) For T-junction power divider of $Z_0 = 50\,\Omega$, determine

Z_1, Z_2 for P_{in} divided in the ratio 1 : 2). Assume $B = 0$



$$P_{in} = \frac{1}{2}\frac{V_0^2}{Z_0}$$

from $P_1 = \frac{1}{2}\frac{V_0^2}{Z_1} = \frac{1}{3}P_{in}$,

$$P_2 = \frac{1}{2}\frac{V_0^2}{Z_2} = \frac{2}{3}P_{in}$$

$$Z_1 = 3Z_0 = 150\,\Omega \;,\; Z_2 = \frac{3}{2}Z_0 = 75\,\Omega$$

$Z_{in} = 75 \,//\, 150 = 50\,\Omega$ Which matches with Z_0

looking from $150\,\Omega$ line $\quad Z_{in} = 50 \,//\, 75 = 30\,\Omega$

Looking from $75\,\Omega$ line $\quad Z_{in} = 50 \,//\, 150 = 37.5\,\Omega$

There is no matching, looking from the output port

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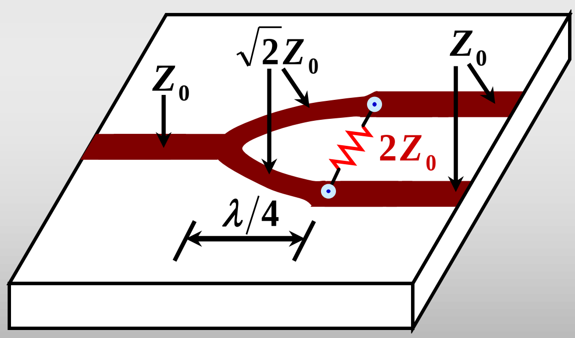
7.3 Wilkinson Power Divider

- Lossy 3-port network with match at all ports
- When only Output ports are matched, we have lossless operation
- Reflected power dissipated in $2Z_0$ resistor and not delivered to other output port. i.e. $S_{32} = S_{23} = 0$. ($\because$ isolation between Output ports)



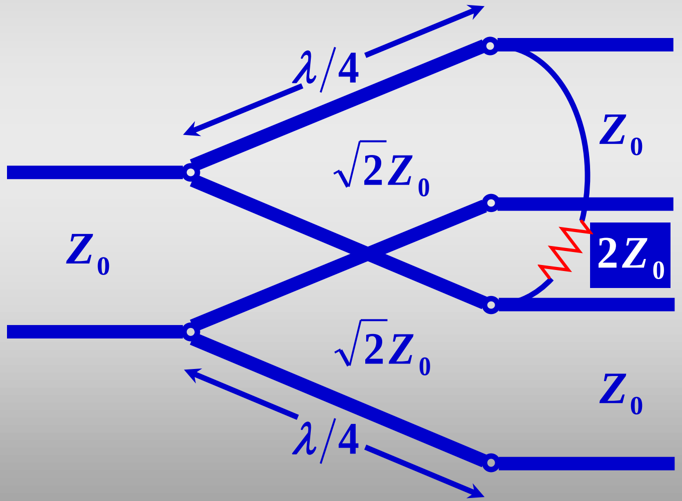


Fig. 7.8 Wilkinson power divider
(a) Microstrip form (b) Equivalent transmission line circuit

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The S parameters of Wilkinson divider are

$S_{11} = 0$ ($Z_{in} = 1$ at port 1)

$S_{22} = S_{33} = 0$ (port 2 and 3 matched for even and odd modes)

$$S_{12} = S_{21} = \frac{V_1^e + V_1^o}{V_2^e + V_2^o} = \frac{-j\sqrt{2}V + 0}{V + V} = \frac{-j\sqrt{2}V}{2V} = -\frac{j}{\sqrt{2}}$$

(symmetry due to reciprocity)

$S_{13} = S_{31} = -\frac{j}{\sqrt{2}}$ (symmetry of ports 2 and 3)

$S_{23} = S_{32} = 0$ (due to short or open at bisection)

7.5 Quadrature (90°) Hybrid

3 dB directional coupler with **90°** phase difference between Output port

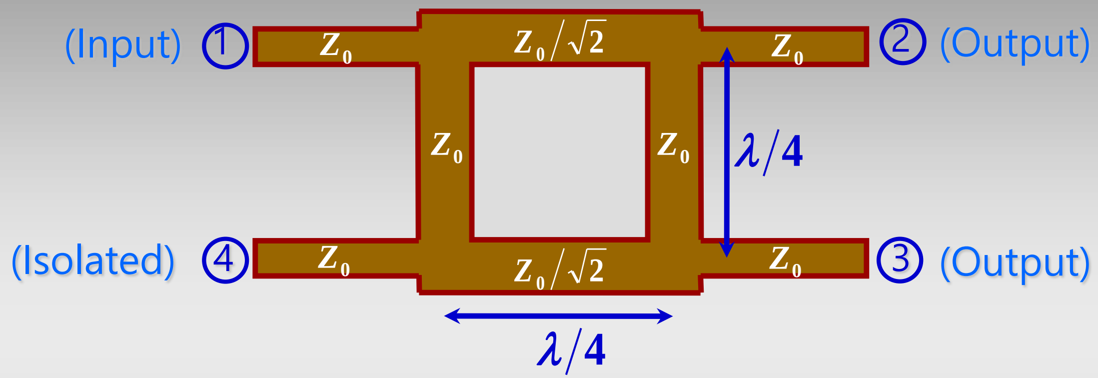
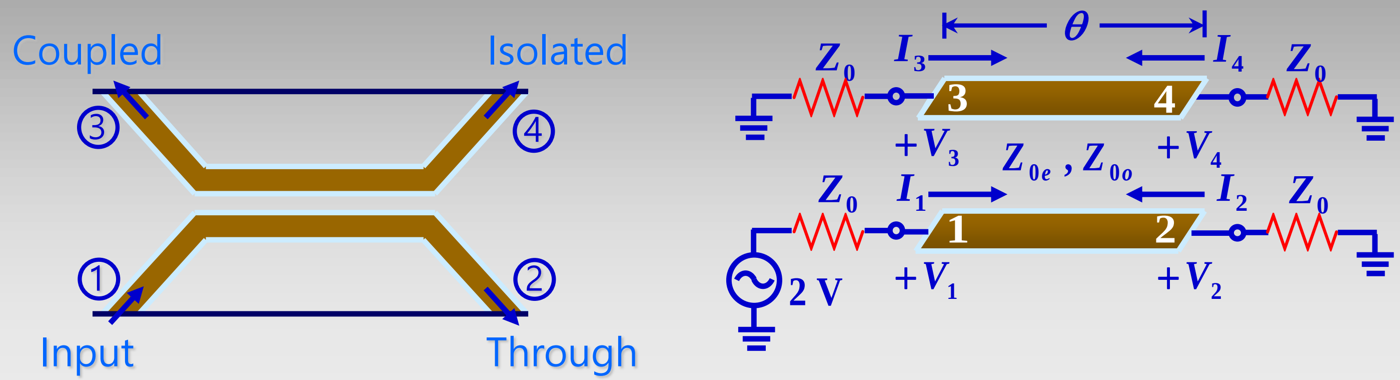


Fig. 7.21 Geometry of a branch-line coupler

$$[S] = \frac{-1}{\sqrt{2}}\begin{bmatrix} 0 & j & 1 & 0 \\ j & 0 & 0 & 1 \\ 1 & 0 & 0 & j \\ 0 & 1 & j & 0 \end{bmatrix}$$

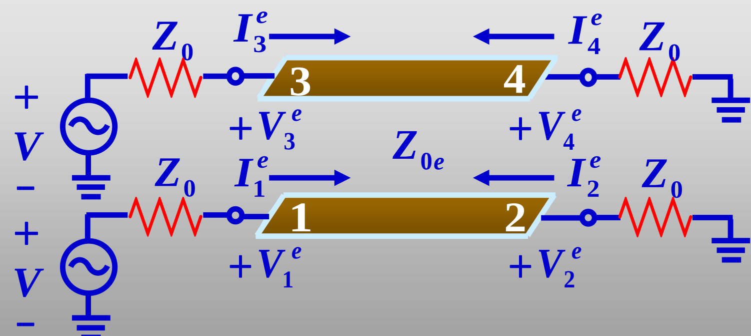
- All port matched
- Input at Port 1, equal division between port 2 and port 3 with **90°** phase difference
- Port 4 is isolated.
- Always symmetric, irrespective of the input.

Design of Coupled Line Coupler



(a) Geometry and port designations (b) The schematic circuit

Fig. 7.31 A single-section coupled line coupler



$$I_1^e = I_3^e \quad , \quad I_4^e = I_2^e$$

$$V_1^e = V_3^e \quad , \quad V_4^e = V_2^e$$

Fig. 7.32 (a) Decomposition of the coupled line coupler circuit of Fig. 7.31 into even mode

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7.8 180° Hybrid

4-port network with **180°** phase difference between the output ports

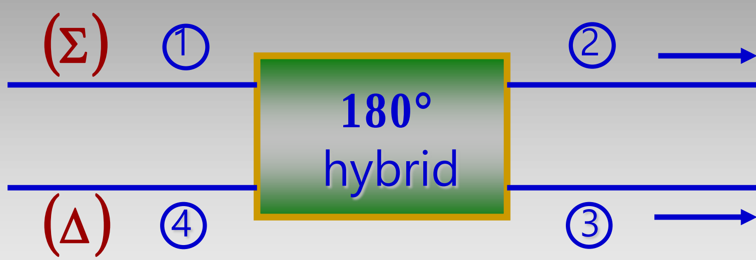
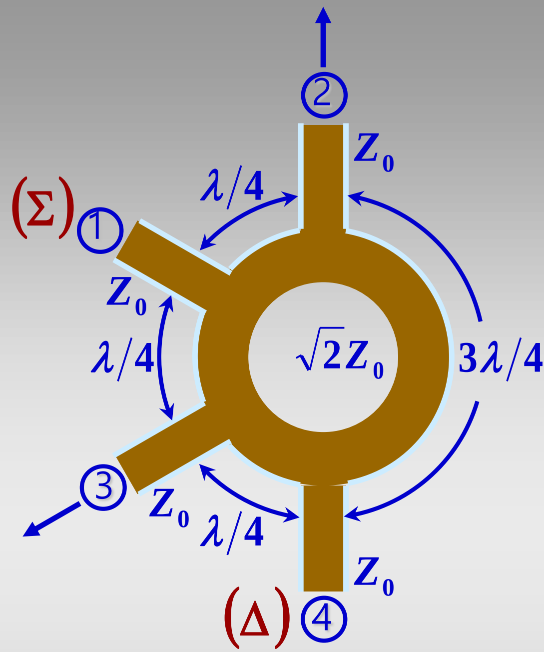


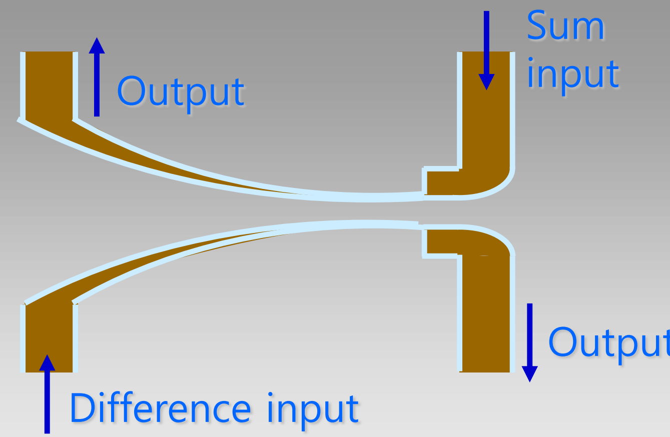
Fig. 7.41 Symbol for a 180° hybrid

$$[S] = \frac{-j}{\sqrt{2}} \begin{bmatrix} 0 & 1 & 1 & 0 \\ 1 & 0 & 0 & -1 \\ 1 & 0 & 0 & 1 \\ 0 & -1 & 1 & 0 \end{bmatrix}$$

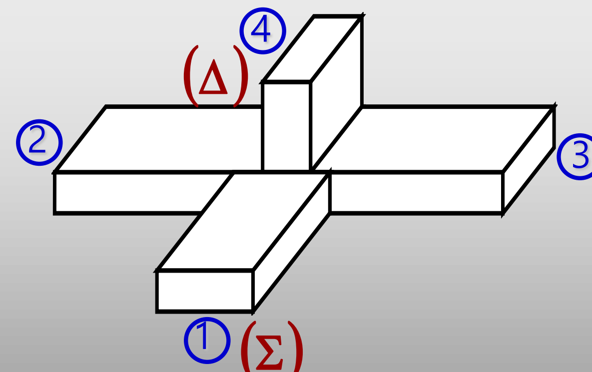
- With input at port 1, inphase output divided into ports 2 and 3, port 4 is isolated
- With input at port 4, 180° phase difference output divided into ports 2 and 3, port 1 is isolated.
- When operating as a combiner, with inputs at ports 2 and 3, sum of inputs appear at port 1 and difference of input at port 4



(a) A ring hybrid (rat-race)
in microstrip or stripline form



(b) A tapered coupled line hybrid



(c) A waveguide hybrid junction (magic-T)

Fig. 7.43 Hybrid junctions

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