CHAPTER 2

Passive Devices

CHAPTER OUTLINE

2.1 IMPEDANCES

Passive devices include resistors, capacitors, and inductors. When these are used in a circuit, the most fundamental consideration is their impedance. The impedance of resistor, capacitor and inductor, are respectively expressed as below:

$$Z_R = R \tag{2.1}$$

$$Z_C = \frac{1}{j\omega C} \tag{2.2}$$

$$Z_L = j\omega L \tag{2.3}$$

Figure 2.1 shows the plot of the magnitude of the impedances on a log-log scale using the above equations. In the case of a resistor, it exhibits constant impedance independent of frequency while the impedance of a capacitor and inductor is seen to decrease and increase linearly with frequency, respectively as shown in Fig. 2.1. However, there are various ways of implementing these devices, which produces similar impedance characteristics for some ranges of frequency. We will thus find out how they are classified, and we will need to look at the changes in the impedance characteristics depending on fabrication methods. In addition, commercially available resistors, capacitors and inductors, may not have the above simple and ideal characteristics in Fig. 2.1. The impedance characteristic generally becomes more complicated as frequency increases, and it is necessary to examine their characteristics in advance during circuit design. At low frequencies, these commercially available resistors, capacitors and inductors generally exhibit the impedance characteristics shown in Fig. 2.1; however a complicated equivalent circuit is needed in practice for their representation as the frequency increases, which adds to the difficulty and complexity of circuit design at high frequencies.

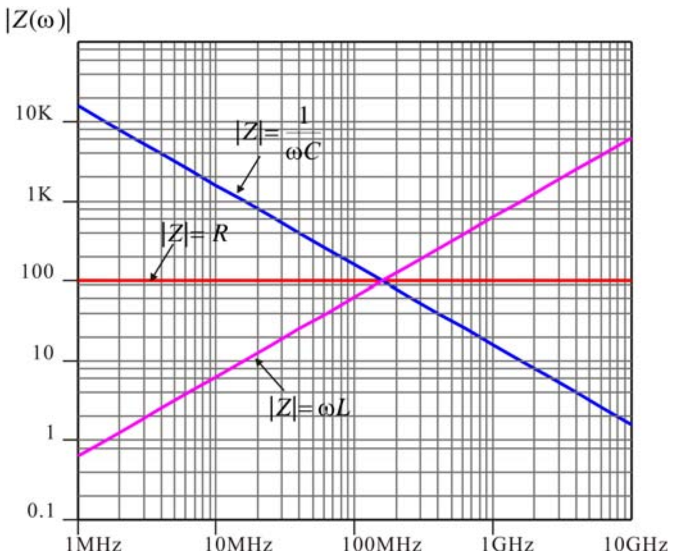


Figure 2.1 Impedances of resistor, capacitor and inductor

In such circumstances, the equivalent circuit that properly reflects these characteristics in the frequency band of operation should be used in the circuit design. The basic thing will be the understanding of typical equivalent circuit of such commercially available resistors, capacitors and inductors, and the technique to determine the equivalent circuit values with a given data. Typical forms of data are available as a datasheet and library in design software. When data are not available, measurements or EM simulations can be carried out to determine them. Therefore in this chapter, detailed data analysis techniques and the measurement techniques in the absence of data will be discussed.

2.2 CLASSIFICATION OF PASSIVE DEVICES

Passive devices are largely classified based on the fabrication method, and are classified into *lead-type* and *chip-type* components, and *pattern-type* passive components. Pattern type passive components are formed by patterns on substrate. Lead-type and chip-type components are widely available commercially; and photographs of these are shown in Fig. 2.2 and Fig. 2.3 respectively.

The assembly method of lead-type and chip-type components differs; while lead-type components are assembled by *insertion technique*, chip-type components are assembled by *surface mounting technique*. In assembling lead-type component, the lead terminals are first bent and inserted into through-holes formed in the printed circuit board. The unnecessary lead terminals are then cut, and the printed circuit board with lead type components is dipped into melt solder, and finally the lead terminals are soldered to the round conductor patterns formed around the through-holes. In the case of chip-type components, *solder creams* are first printed to *land-patterns* formed on a printed circuit board or substrate, and chip-type components are mounted on the land patterns manually or automatically. Finally passing through *reflow machine* with appropriate temperature profile, soldering chip-type components to the land-patterns is completed. In the case of lead-type-components, it should be noted that the lead terminals are supposed to be used for only connection. However, they also give rise to parasitic inductance at high frequencies, and consequently this adds to the impedance of the passive device. These lead-type components were mostly used as passive devices in the past before chip-type components became commercially popular. But due to the fluctuations arising from the length of the lead terminals when assembled, it is seldom used at high frequency although it is still used even to date. Compared with the lead-type components, chip-type components obviously give rise to smaller parasitic circuit elements. Furthermore, lead-type components are generally bigger than chip-type components, which is another disadvantage in terms of the size.

(a) (b) (c)

Figure 2.2 Lead-type components; (a) resistor, (b) capacitor, and (c) inductor

(a) (b) (c)

Figure 2.3 Chip-type components; (a) resistor, (b) capacitor, and (c) inductor

Chip-type components compared to lead-type components have relatively less parasitic elements caused by the terminals, which becomes profitable at high frequency. However, the key reason for the popularity of these devices is miniaturization rather than their advantage for use at high frequency. In addition, surface-mounting technique which provides an advantage for large-scale fabrication is another reason why they are being widely used.

Pattern-formed passive components can be produced on the substrate usually by the process of Monolithic Microwave Integrated Circuit (MMIC) or thin film. In situations where a circuit designer directly designs passive devices by pattern formation, there is limitation in the range of values attainable compared to chip-type or lead type devices, which is a disadvantage.

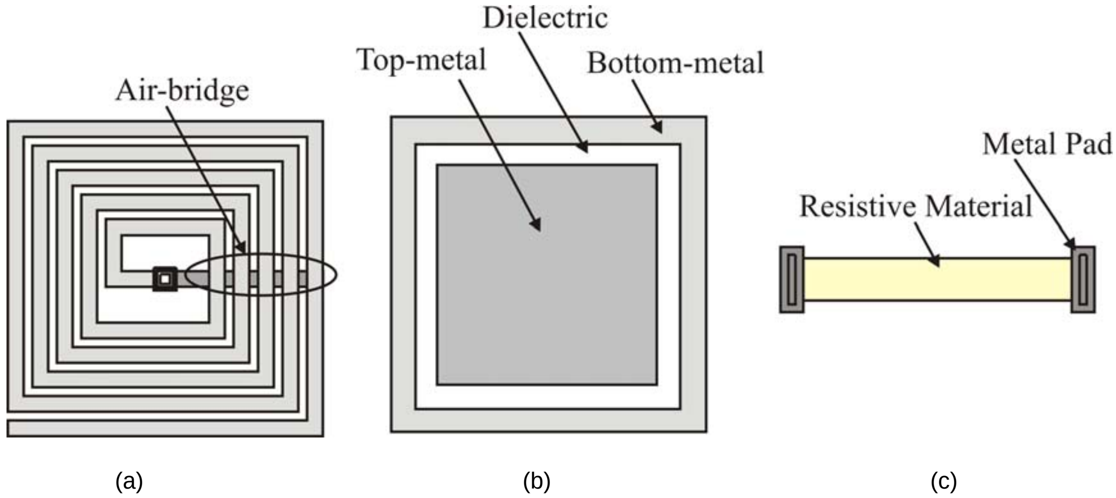


Figure 2.4 Passive components produced by pattern formation: (a) inductor, (b) capacitor, and (c) resistor

In the case of a resistor as shown in Fig. 2.4(c), a thin film of a resistive material (usually NiCr and TaN) forms a resistor pattern, on which the conductor pattern (for the purpose of connection) is produced by proper technique. The resistor in Fig. 2.4(c) has a uniform thickness, and the resistance is thus proportional to the length L and inversely proportional to the width (W). The proportionality constant is defined as the *sheet resistivity* R_S, and the resistance R is given by

$$R = R_S \frac{L}{W} \tag{2.4}$$

Note that R_S has the dimension of ohms/square. In the case of a capacitor in Fig. 2.4(b), a uniform thickness dielectric sheet is formed on the bottom conductor pattern, and a top conductor pattern is again formed on the dielectric, and a Metal-Insulator-Metal (MIM) capacitor is formed. At this point the dielectric material's thickness, generally determined by the process, is constant; and once the *sheet capacitance* C_S has been determined, the capacitance is then directly proportional to the surface area (A) and is expressed as:

$$C = C_S A \tag{2.5}$$

Certainly, as the frequency increases, the parasitic elements inherent in this capacitor will appear, and needs to be adequately modeled for in the frequency band of operation.

In the case of an inductor, it is usually implemented as a spiral type shown in Fig. 2.4(a) above. Its design is not as simple as that of capacitor or resistor. Foundry service companies usually provide measured results or data on several configurations of inductors; and in the absence of those, the values of inductors are determined through Electro-Magnetic (EM) simulation. In addition, although the assessment of this type of pattern-type components is important, when viewed from design point of view, the construction of their equivalent circuit is the same as that of chip-type or lead-type components, and so its separate discussion will be omitted here.

■ Example 2.1

(1) Given that the sheet resistance R_S is 50 ohm/square meter, calculate the resistance of the component shown in Figure 2E.1 below.
(2) In addition, the permittivity of a capacitor is 7.2 and its thickness is 0.4 μm; determine the sheet capacitance in pF/mm^2 and hence calculate the capacitance of a 50 μm^2 capacitor.

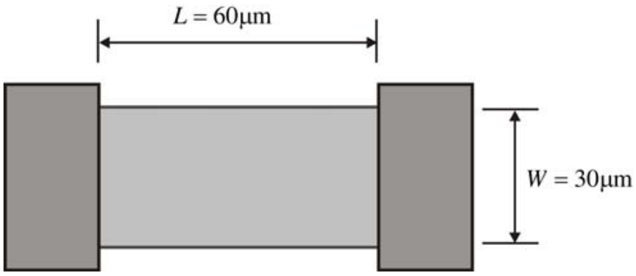


Figure 2E.1 Resistor

Solution

(1) In the case of the resistor, since the value of the sheet resistance R_S is 50 ohm/square meter, then.

$$R = R_S \frac{L}{W} = 50 \times \frac{60}{30} = 100\Omega$$

(2) In the case of the capacitor, the sheet capacitance per square mm is

$$C_S = \varepsilon_r \varepsilon_0 \frac{A}{t} = 8.854 \text{ pF/m } \times 7.2 \times \frac{1 \text{ mm}^2}{0.4 \ \mu\text{m}}$$
$$= 8.854 \text{ pF/m } \times 7.2 \times \frac{1 \text{ m}}{0.4} = 159 \text{ pF}$$

Thus, C_S = 159 pF/mm^2 and for a capacitor having an area of 50 μm × 50 μm, the capacitance is

$$C = C_S \times 0.05^2 = 159 \times 0.05^2 \text{ pF=0.398 pF}$$

■

2.3 EQUIVALENT CIRCUIT OF CHIP-TYPE PASSIVE DEVICES

As we have seen, in cases where thin film or MMIC process is not used, the most likely choice for passive components in high frequency application is chip-type components and thus they are widely used. Therefore, in this chapter, chip-type device manufacturing and evaluation method will be explained. In addition, the method of extracting the equivalent circuit from given data will be explained.

2.3.1 Chip-type Capacitors

The chip capacitor is usually constructed in a multi-layer structure as shown in Fig. 2.5. The two solder terminals are connected in parallel to a number of conducting plates (in Fig. 2.5, they are named as internal electrodes); and the sum of the capacitance formed between the conducting plates appears at the terminals of the capacitor. The dielectric fills the space between the conducting plates and its dielectric constant of further increases the capacitance between the terminals.

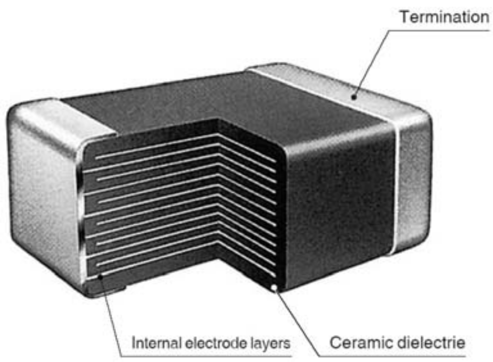


Figure 2.5 Structure of a chip capacitor

By way of specification, the chip capacitors are identified based upon the geometrical parameters shown in Fig. 2.6. They are identified based on two geometrical parameters; length *L* between the terminals and terminal width, *W*. Based on a standard unit of mm; a capacitor having a length of 1.0 mm and a width of 0.5 mm, is called type 1005; and following a similar definition, type 1608 is a capacitor which has a length of 1.6 mm and a width of 0.8 mm. This classification is not only limited to capacitors but also resistors or inductors are identified in the same way; a 1608 resistor similarly represents a chip resistor with a dimension of 1.6 mm length and 0.8 mm width.

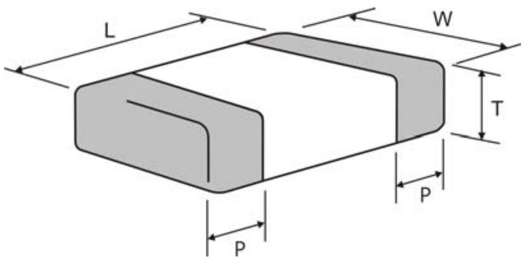


Figure 2.6 Dimensions of a chip-type component

The impedance of such a capacitor depends on the frequency and the general equivalent circuit is as shown in Fig. 2.7. In this circuit, *C* represents the capacitance of a chip capacitor, while *L* is the parasitic inductance that appears due to the multilayer structure and *R* represents the loss in the conducting material used in the capacitor. Therefore, from the structural point of view, as the size of the capacitor becomes smaller, inductance *L* generally becomes smaller, and the inductance generally tends to be constant regardless of the value of the capacitance.

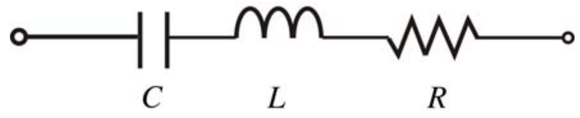


Figure 2.7 The equivalent circuit of a chip capacitor

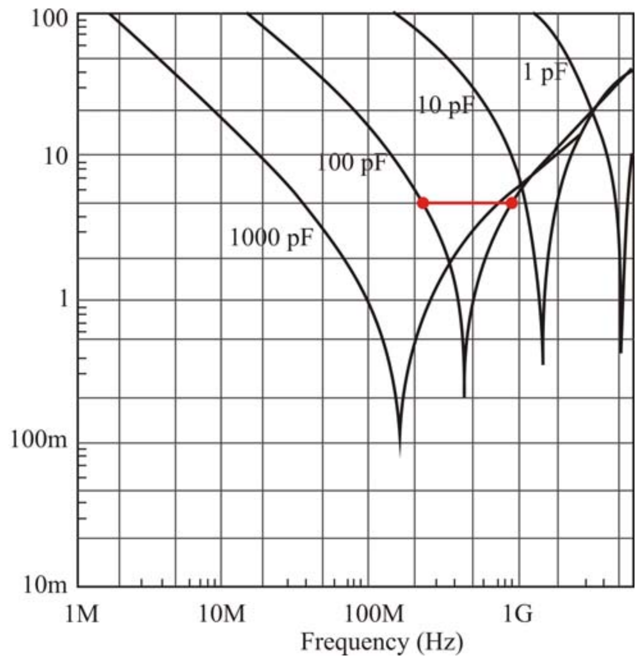


Figure 2.8 Impedance characteristics with frequency of chip capacitors[1]

Figure 2.8 shows the impedance of such a capacitor with respect to frequency. Looking at the 100 pF curve in Fig. 2.8, below about a frequency of 100 MHz, it can be seen that the impedance decreases linearly with frequency. This is obvious because at low frequency, the chip capacitor behaves as an ideal capacitor. Since it shows about an impedance of 20 ohms at 70 MHz, it can be seen to have a capacitance of

$$C = \frac{1}{2\pi f X_C} \cong \frac{1}{2\pi \times 70 \times 10^6 \times 20} = \frac{1000}{2\pi \times 1.4} \text{ pF} \cong 113 \text{ pF}$$

The error in the calculated capacitance is due to the approximate reading of the impedance value from the graph. In addition, as the frequency increases, the impedance after reaching a minimum rises again. This can be understood from the equivalent circuit of the capacitor in Fig. 2.7. While the impedance of the capacitor is decreasing, that of the inductor is increasing with frequency. Thus

[1] Murat Manufacturing Co. Ltd., Chip Monolithic Ceramic Capacitor, 1999

as the frequency increases to higher frequency, the capacitor behaves as an inductor, which explains why the impedance increases with frequency.

Considering the minimum point of the 100 pF curve in Fig. 2.8, the resistor in the equivalent circuit of Fig. 2.7 is found to have a series resistance value of approximately R= 0.2 ohm. Furthermore, assuming that the impedance is mainly by an inductor at a frequency of 1 GHz, the approximate value of the inductor can be obtained from the curve. Since its impedance is approximately 6 ohm at this frequency, the approximate inductance value is found to be

$$L = \frac{X_L}{2\pi f} \cong \frac{6}{2\pi \times 1 \times 10^9} = \frac{6}{2\pi} \text{ nH} \cong 1 \text{ nH}$$

More accurate value of the inductance can be determined by curve fitting, which is widely used to fit such data.

Thus, in the case of the 100 pF capacitor, when used at a frequency of over 1 GHz it is closer to being an inductor rather than a capacitor. If used as a DC block, it must have an impedance of about 5 ohm (which is estimated as 1/10 of a standard impedance value of 50 ohm). From the graph of Fig. 2.8, it is possible to use this capacitor as a DC block or a bypass capacitor between the frequencies of 300 MHz to 900 MHz. Its use as DC block at a higher frequency can be found to pose a difficulty due to the effect of the parasitic inductor.

2.3.2 Chip-type Inductors

The manufacturing methods of chip-type inductors are somewhat various compared with chip type capacitors. However, inductor is basically formed by winding *enamel coated copper wire* on a ferrite core material as shown in Fig. 2.9 and by forming appropriate solder terminals for connection. More winding is possible when the wire of thinner diameter is used, which results in higher inductance. The inductance can be significantly increased by increasing the number of windings. It should however be noted that, this will lead to increasing series resistance and decreasing current capacity. In addition, this will also lead to a proportional increase of parasitic capacitance. Therefore, as the inductance becomes larger, they usually cannot be used at high frequency. Thus, when inductors are employed in designing a circuit, the datasheet should be carefully referred to verify the frequency range of operation.

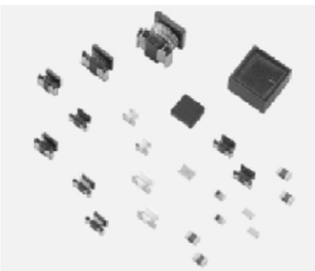


Figure 2.9 Photograph of chip inductors

Furthermore, when such an inductor is viewed as a resonator, generally its Q is low. Thus, when applied in oscillator design as a resonator, a close examination is required. Figure 2.10 is a typical electrical equivalent circuit of an inductor; where L represents the inductance arising from the winding, R represents the winding resistance, and C represents the sum of parasitic capacitances appearing between the windings.

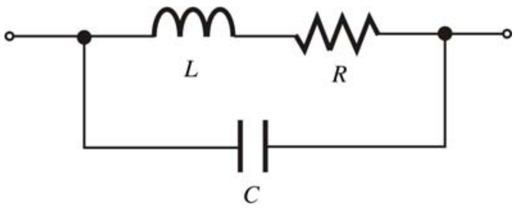


Figure 2.10 Equivalent circuit of an inductor

At extremely low frequency, the inductor usually behaves like a resistor. As the frequency becomes higher, the impedance due to the inductance arising from the winding becomes dominant and then it behaves as an inductor. As the frequency becomes much higher, the impedance of the capacitor connected in parallel becomes smaller and then the inductor behaves as a capacitor. Thus the approximate frequency range for using an inductor is usually:

$$\frac{R}{L} < \omega < \frac{1}{\sqrt{LC}} \tag{2.6}$$

Figure 2.11 shows the impedances of chip-type inductors with respect to frequency. In this figure, examining the 100 μH curve, it is found that, the impedance increases linearly with frequency up to 10 MHz. Thus below 10 MHz it behaves as an inductor. Since the impedance value at 1 MHz appears to be approximately 1 kohm, the inductance is found to be

$$L = \frac{X_L}{2\pi f} \cong \frac{1\times 10^3}{2\pi \times 1\times 10^6} = \frac{1}{2\pi}\ \text{mH} = 159\ \mu\text{H}$$

Due to the approximate reading of the impedance value from the graph, an error arises in the value of the inductance. Furthermore, as the frequency increases, the impedance begins to fall after reaching a maximum point; which is due to the influence of the parasitic capacitor in the equivalent circuit. Thus as the frequency becomes much higher, the inductor acts as a capacitor. In addition, assuming that at a frequency of 100 MHz the impedance is mainly due to a capacitor; then since its impedance is approximately 1 kohm at this frequency, the value of the capacitor is found to be:

$$C = \frac{1}{2\pi f X_C} \cong \frac{1}{2\pi \times 1\times 10^3 \times 100\times 10^6} = \frac{10}{2\pi}\ \text{pF} \cong 1.59\ \text{pF}$$

Furthermore, since the resistance is 100 kohm at the maximum point in Fig. 2.11, then the value of the series resistor in the equivalent circuit becomes

$$\frac{(\omega_o L)^2}{R} = 100\ \text{K}$$

and from this a value of

$$R = \frac{(\omega_o L)^2}{100\ \text{K}} = \frac{(2\pi \times 10 \times 10^6 \times 100 \times 10^{-6})^2}{100 \times 10^3} = 4\pi^2 \times 10 \cong 390\Omega$$

is obtained.

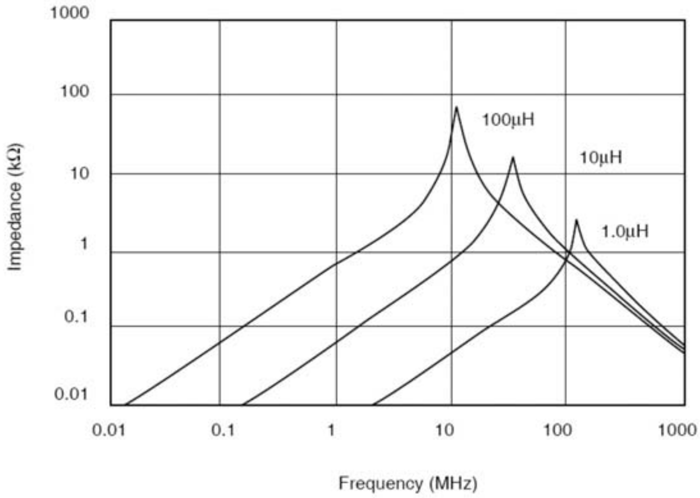


Figure 2.11 Impedance characteristics of chip inductors with frequency[2]

2.3.3 Chip-type Resistors

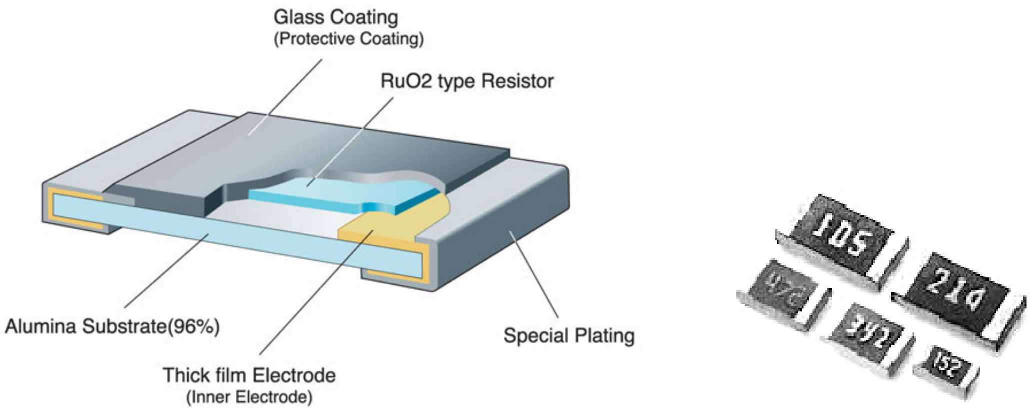
The structure of chip-type resistor is shown in Fig. 2.12. As you can see in the figure, chip-type resistor is manufactured by printing a resistive material (RuO_2) on a ceramic substrate and after the conductor pattern has been printed (Thick-film electrode in Fig. 2.12), terminals are plated to make it possible for soldering. Furthermore, in order to prevent oxidation or damage to the resistive material; a glassy material is coated on the resistive material in a post-processing.

A similar specification to that of capacitors (1005, 1608, and 2010) is used in the identification of chip resistors too. As the size gets smaller, similar to a capacitor, the impedance characteristics of chip resistors generally become more ideal, i.e, can be applied to higher frequency. Another thing that needs to be noted is the power dissipation. The power consumption that such a resistor can withstand has been usually listed in the manufacturers' data sheet. The power consumption capability becomes generally smaller as the size shrinks.

Depending on the manufacturer, the value of the resistor is marked on its surface as shown in Fig. 2.12(b) which makes the identification of the value of the resistor much easier. In Fig. 2.12, following a general notation, the first two digits represent the effective value of the resistance and the remaining digits represent the exponent; thus a 3.4k resistor is denoted as:

[2] Murata Manufacturing Co. Ltd., Chip Coil, O05E6.pdf 01.5.9, 2001

$$342 = 34\times10^2 = 3.4\times10^3 = 3.4\ \text{k}$$



(a) (b)

Figure 2.12 Chip-type resistor; (a) structure and (b) photo

The frequency characteristics and equivalent circuit of a chip resistor is generally not known. The method explained in the following section can be used for the chip-type resistors or components whose frequency response of the impedance is unknown. Using the measured frequency response of the impedance, the equivalent circuit may be found using the previously explained method.

2.4 MEASUREMENT OF IMPEDANCE CHARACTERISTICS

Chip-type passive components can all be considered as 1-port components. Thus, using an *impedance analyzer* or a *network analyzer*, their impedance characteristics can be measured through a 1-port measurement. Here, the method of measuring the impedance characteristics using the widely available network analyzer is presented. It should be noted that the method presented here may accompany the significant errors in high frequency.

Firstly, in order to perform measurement using network analyzer, a coaxial *Small Miniature Assembly* (SMA) connector must be prepared and its soldering tab (if present) should be removed as in Fig. 2.13. Next, you would see that the normal reference plane is plane, T in Fig. 2.13 after completing 1-port calibration. Here, you should understand for the time being that the calibration means moving the measurement reference plane to plane T and elimination of the non-ideal characteristics of the equipment. Thus, the impedance at plane T can be correctly measured through the calibration without the effects of cables and adapters that can possibly be used in the connection. Generally, in practical measurement, cables and adapters can be included for connection. They shift the reference plane and affect the measurement due to its non-ideal characteristics. These effects are removed through the calibration. After the calibration, the reference plane is usually defined at plane T of the connector.

The measured results include the used SMA connector but the correct impedance of passive component is defined at plane T'. Therefore the length of the used SMA connector should be removed from the measured results and this can usually be done by using the *electrical delay* or *port extension* function incorporated in the network analyzer. Since plane T' in Fig. 2.13 would present an open circuit in the measurement when the passive component removed, (and this represents an infinite impedance if there is nothing connecting to point T') the measured impedance is changed to show open circuit impedance through the adjustment of the electrical delay. Consequently, the measurement will be accompanied by some errors but the measured impedance within the frequency band will show the open circuit at point T'. In other words, the calibrated reference plane is moved to T' through an electrical delay.

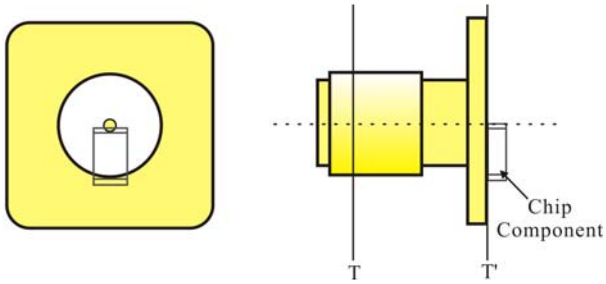


Figure 2.13 Assembly for the measurement of passive component; (a) back and (b) side views.

Next, the passive *device under test* (DUT) is connected by soldering it to a connector as shown in Fig. 2.13. In this state, the measured result is the impedance value of the device. Figure 2.14 shows the impedance characteristics of a passive device measured using this method. Here, the device under test is a varactor diode acting as a variable capacitor. As can be seen from Fig. 2.14, it will be noted that the reactance increases with frequency from a negative value, passing through the 0 point and continuing to increase in the positive direction.

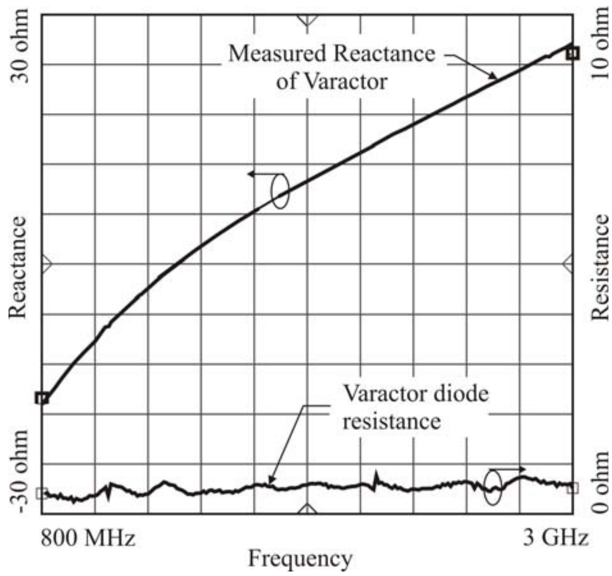


Figure 2.14 Measurement result for chip varactor diode using the proposed method.

It can be seen that the series resistance of this varactor diode is approximately 0.5 ohm; the reactance is also as that of a capacitor at low frequency; however as frequency further increase, it will be seen to appear as an inductor. With this result, the values of the components in the equivalent circuit of Fig. 2.7 can be determined through optimization. The limitations of the proposed measurement method will primarily be in the connector. Typical SMA connectors can be used up to 18 GHz. Thus, when using SMA connectors for connection, measurement of impedance characteristics is possible only up to below 18 GHz. However, even the use of coaxial connectors (to be described later; 2.9 mm or 2.4 mm coaxial connector) which can be used up to higher frequencies, instead of the SMA, does not improve the accuracy of the measurement at higher frequency. The reason for this is primarily, as shown in Fig. 2.13, an open circuit is created by the open coaxial connector; and in practice the fringing capacitance arising from the open-end of the connector has to be calibrated. Furthermore, the resistance arising from radiation should be considered and corrected in order to obtain accurate results. In addition, no matter how this gets corrected, when a device is attached, the effect of the coaxial connector distorts the shape of the field; which is the reason behind the difficulty in obtaining accurate results for devices at higher frequency.

■ Example 2.2

Open the Murata capacitor library in ADS and after measuring the 10 pF impedance by simulation; obtain its equivalent circuit.

Solution

In the schematic of Fig. 2E.2, **Vout** becomes the impedance since AC current source is set to1A.

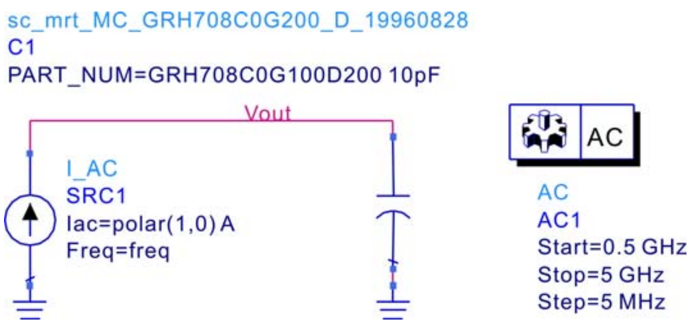


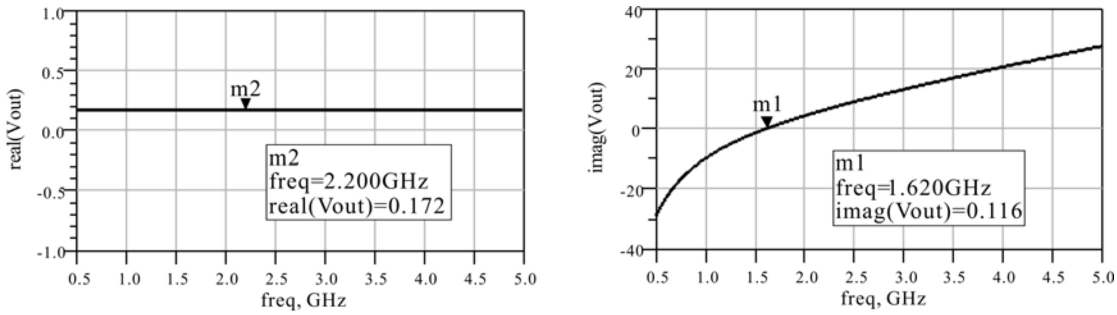
Figure 2E.2 Measurement of the impedance of a capacitor

Plotting the real and imaginary parts of **Vout** separately, the graphs in Fig. 2E.3 are obtained; where the value of the real part is read as R =0.17Ω.

Furthermore, in Fig. 2E.3(b), the imaginary part can be seen to represent a series resonance. Thus the slope near the resonant frequency becomes

$$\left.\frac{\partial X}{\partial f}\right|_{f_o} = 2\pi \left.\frac{\partial X}{\partial \omega}\right|_{f_o} = 2\pi \frac{\partial}{\partial \omega}\left.\left(\omega L - \frac{1}{\omega C}\right)\right|_{f_o} = 4\pi L$$

This means the inductance of series resonant circuit can be obtained through the slope of the reactance at the resonance frequency. Plot of the imaginary part near the resonant frequency is presented in Fig. 2E.4. To compute the slope, markers are inserted to the plot.



(a) (b)

Figure 2E.3 (a) Real and (b) imaginary parts of the impedance.

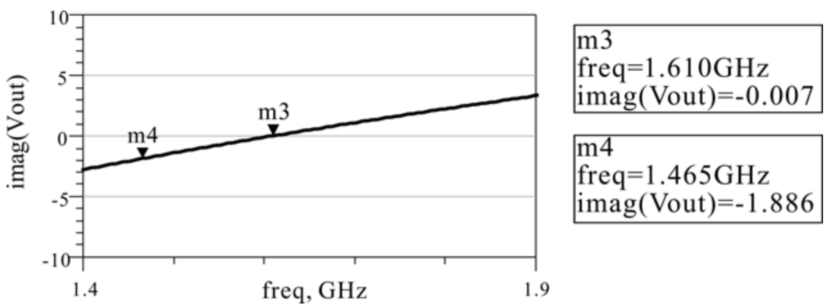


Figure 2E.4 The imaginary part of the impedance around the resonant frequency

The equations to calculate the slope at the resonance frequency using the marker values is shown in Fig. 2E.5. In Fig. 2E.5, **m**3 and **m**4 represent the *y* values (vertical) of the displayed markers. Also **indep**(**m**3) and **indep**(**m**4) are the *x* (horizontal) values of the displayed markers. Thus, the first equation in Fig. 2E.5 becomes the slope divided by 4π, which corresponds to the inductance calculated from $L=\partial X/\partial f/(4\pi)$. The second equation is the capacitance calculated from the determined value of *L* and the resonant frequency because $C=1/(\omega_o)^2L$. The computed values using the equation in Fig. 2E.5 are $R = 0.17\ \Omega$, $L = 0.977$ nH and $C = 10$ pF. Thus the equivalent circuit of the chip capacitor is obtained. Finally, the last equation is the reactance **X** calculated from the obtained values of *L* and *C*. The reactance **X** gives the verification of the computed *L* and *C* values. If the computed *L* and *C* values are close to fit the imaginary part, **X** will show good agreement. The comparison is shown in Fig. 2E.6 and a very good agreement can be seen. Therefore it can be found the equivalent circuit produces the reactance close to the imaginary part obtained from the simulation.

```
Eqn L=(m3-m4)/(indep(m3)-indep(m4))/(4*pi)
Eqn C=1/((2*pi*indep(m3))**2*L)
Eqn X=2*pi*freq*L-1/(2*pi*freq*C)
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Figure 2E.5 Equations for the calculation of the equivalent circuit values in the display window.

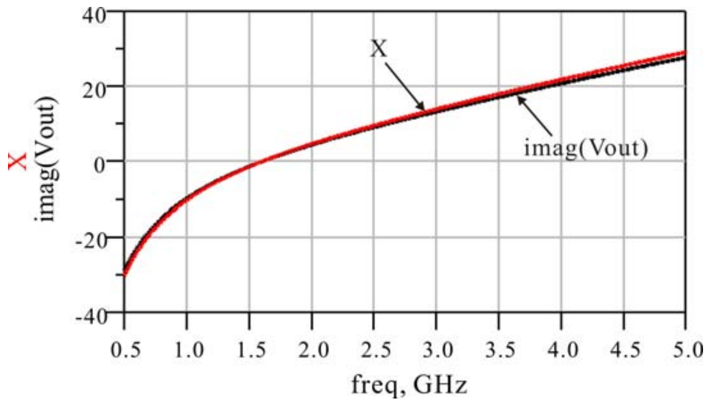

Figure 2E.6 Comparison of the imaginary parts of the impedances obtained from calculation and from simulation.

■

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[2] GEC Marconi, *GaAs IC foundry design manual*, Oct., 1997.

[3] I. D. Robertson ed., *MMIC Design*, The Institution of Electrical Engineers, London, 1995

PROBLEMS

2.1 A company A fabricates a thin film resistor whose sheet resistivity is 50 ohm/square and in the case of a company B, an identical thin film resistor has a sheet resistivity of 100 ohm/square. What is the difference between the processes of the two companies? Furthermore, given that the material's volume resistivity is ρ and its thickness is t, find its sheet resistivity.

2.2 Given that the sheet resistivity is 50 ohm/square meter and we want to design a 100 ohm resistor. If the current flowing in this resistor is 2 mA, find its minimum width. Its rated current per width is 0.5 mA/μm.

2.3 The dielectric material of a MIM capacitor is Silicon Nitride and its dielectric constant is 7.2. If the area is 30 μm square find the thickness of a 0.53 pF capacitor. In addition find the capacitance per unit area.

2.4 In the equivalent circuit of an inductor such as that in Fig. 2.10, show that the approximate magnitude of the impedance at resonance is

$$|Z_{\max}| = \frac{(\omega_o L)^2}{R}$$
$$\omega_o = 1/\sqrt{LC}$$

2.5 In this text, we have covered the method of extracting the equivalent circuit of a capacitor from $|Z(\omega)|$ characteristics obtained from impedance analyzer such as that shown in Fig. 2.8. The impedance of the equivalent circuit of a capacitor is

$$Z(\omega) = R + jX = R + j\left(\omega L - \frac{1}{\omega C}\right)$$

Using this, the real and imaginary parts for frequency can be obtained by graphical representation. Then show that the approximate impedance near the resonant frequency ω_o is

$$Z(\omega) \cong R + j2L(\omega - \omega_o)$$

2.6 Similar to Problem 2.5 above, for a parallel resonant circuit, show that the admittance near the resonant frequency, $Y(\omega)$ is given by

$$Y(\omega) \cong G + j2C(\omega - \omega_o)$$

2.7 (Method of extracting equivalent circuit) from the resonant frequency of Problem 2.5, by using the slope of the reactance versus frequency

$$\left.\frac{\partial X}{\partial f}\right|_{f_o} = 4\pi L$$
$$\omega_o = 1/\sqrt{LC}$$

The values of L and C can be found while R can be found from the real part of the impedance. Use the result in Fig. 2.14 and determine R, L, and C.