
8장 LNA 설계

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Gain & Circles

Thevenin Equivalent Circuit

Z_o ; Reference Impedance

Superposition

$$V = E_o \frac{Z_T}{Z_o + Z_T} + E_T \frac{Z_o}{Z_o + Z_T} = V^+ + V^-$$

and $V^+ = \frac{E_o}{2}$

$$V^- = \frac{E_o}{2} \cdot \frac{Z_T - Z_o}{Z_T + Z_o} + E_T \frac{Z_o}{Z_o + Z_T}$$

$$= V^+ \Gamma_T + E_T \frac{Z_o}{Z_o + Z_T}$$

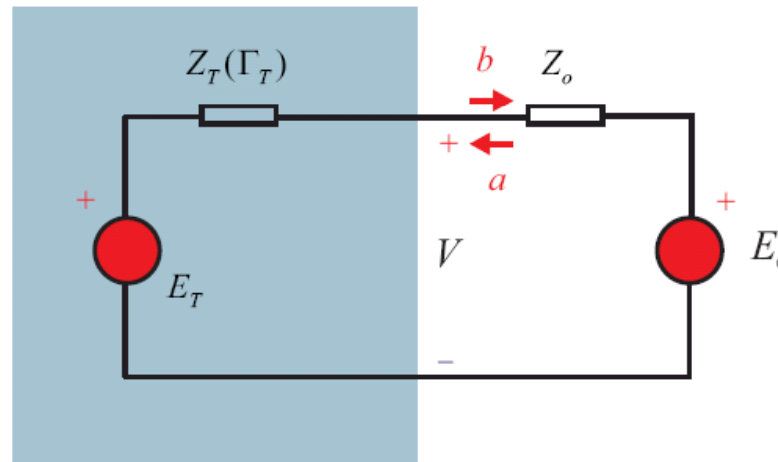
Hence

$$b = \frac{V^-}{\sqrt{2Z_o}} = \frac{V^+}{\sqrt{2Z_o}} \Gamma_T + \frac{E_T}{\sqrt{2Z_o}} \frac{Z_o}{Z_o + Z_T} = \Gamma_T a + b_T$$

where

$$b_T = \frac{E_T}{\sqrt{2}} \frac{\sqrt{Z_o}}{Z_o + Z_T}; \text{ The normalized reflected voltage at the termination } Z_o$$

$$\Gamma_T; \text{ The reflection coefficient with inner source killed}$$



Power Relation

Q1; P_L Delivered Power to load Γ_L ?

$$P_L = |a_L|^2 - |b_L|^2 = |a_L|^2 (1 - |\Gamma_L|^2) \rightarrow a_L ?$$

Q2; P_A Available Power from the source ?

$$P_A = P_L |_{\Gamma_L = \Gamma_s^*}$$

From the circuit

$$b_L = \Gamma_L a_L; \quad b_g = b_s + \Gamma_s a_g$$

and

$$a_L = b_g = b_s + \Gamma_s a_g = b_s + \Gamma_s b_L = b_s + \Gamma_s \Gamma_L a_L$$

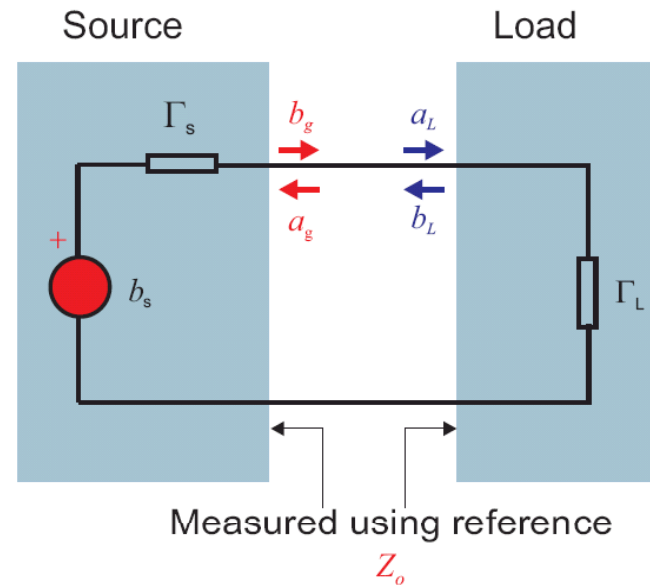
$$\rightarrow a_L = \frac{b_s}{1 - \Gamma_s \Gamma_L}$$

The Delivered Power;

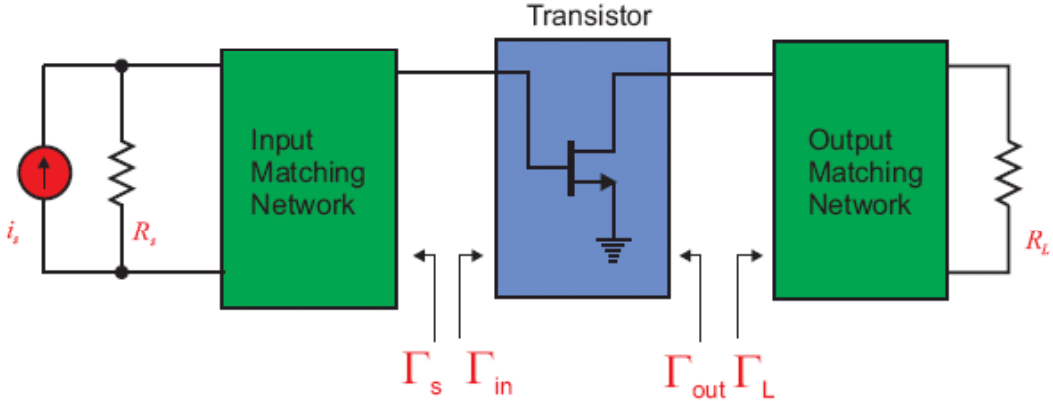
$$P_L = |a_L|^2 - |b_L|^2 = \frac{|b_s|^2 (1 - |\Gamma_L|^2)}{|1 - \Gamma_s \Gamma_L|^2}$$

The Available Power by substituting $\Gamma_s = \Gamma_L^*$;

$$P_A = \frac{|b_s|^2}{1 - |\Gamma_s|^2}$$



Definitions of Reflection Coefficients

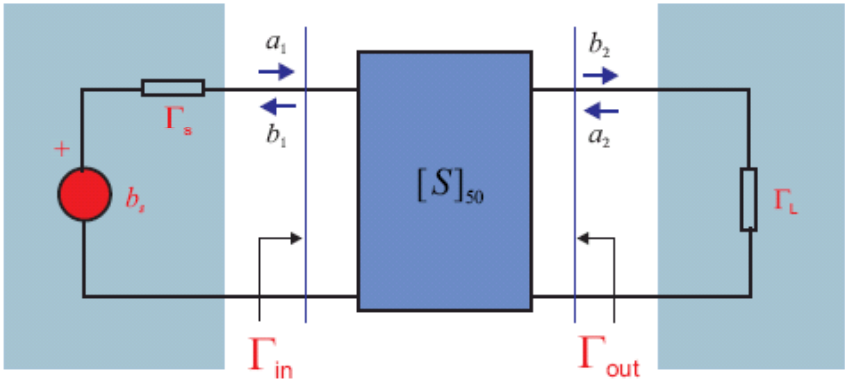


Γ_s ; **Source (Impedance)**

Γ_L ; **Load (Impedance)**

Γ_{in} ; **Input (Impedance)**

$$= S_{11} + \frac{S_{12}S_{21}\Gamma_L}{1 - S_{22}\Gamma_L}$$



Γ_{out} ; **Source (Impedance)**

$$= S_{22} + \frac{S_{12}S_{21}\Gamma_s}{1 - S_{11}\Gamma_s}$$

Note; Input & Output matching networks are lossless

Transducer Power Gain

$$G_T = \frac{\text{Delivered Power to Load}}{\text{Available Power from source}} = \frac{P_L}{P_A}$$

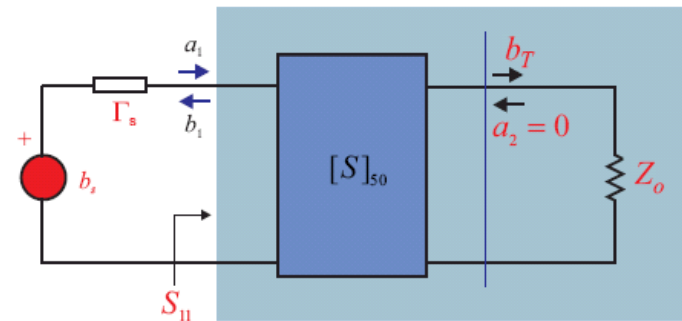
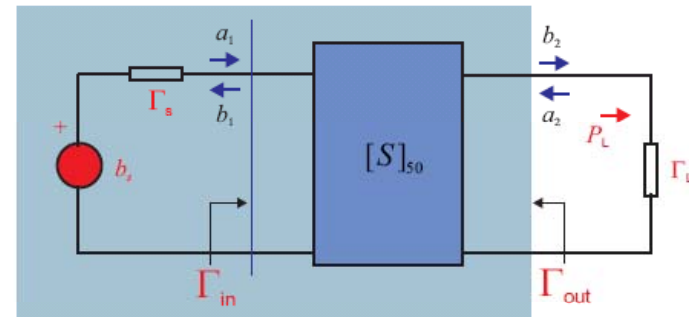
Note that

$$P_A = \frac{|b_s|^2}{1 - |\Gamma_s|^2},$$

and Thevenin equivalent at the load

$$\Gamma_T = \Gamma_{out}$$

$$b_T = b_2|_{a_2=0} = S_{21}a_1 + S_{22}a_2 = S_{21}a_1 = S_{21} \frac{b_s}{1 - S_{11}\Gamma_s}$$



G_T Continued

$$G_T = \frac{\text{Delivered Power to Load}}{\text{Available Power at Input}} = \frac{P_L}{P_A}$$

Available Power

$$P_A = \frac{|b_s|^2}{1 - |\Gamma_s|^2},$$

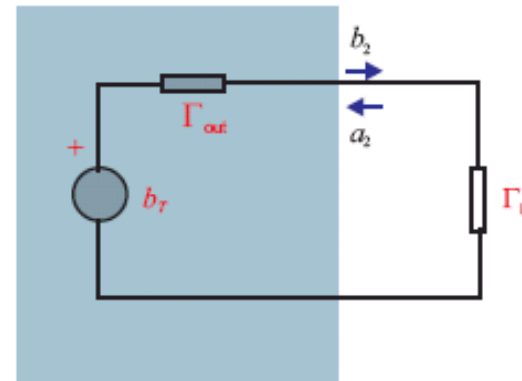
Delivered Power

$$\begin{aligned} P_L &= |b_2|^2 (1 - |\Gamma_L|^2) = \frac{|b_T|^2 (1 - |\Gamma_L|^2)}{|1 - \Gamma_{out} \Gamma_L|^2} \\ &= \frac{|b_s|^2 |S_{21}|^2 (1 - |\Gamma_L|^2)}{|1 - S_{11} \Gamma_s|^2 |1 - \Gamma_{out} \Gamma_L|^2} \end{aligned}$$

Therefore

$$G_T = \frac{P_L}{P_A} = \frac{(1 - |\Gamma_s|^2) |S_{21}|^2 (1 - |\Gamma_L|^2)}{|1 - \Gamma_{out} \Gamma_L|^2 |1 - S_{11} \Gamma_s|^2}$$

Thevenin Equivalent Circuit
at Load plane



Alternative Form of G_T

Note that

$$a_1 = \frac{b_s}{1 - \Gamma_{in}\Gamma_s}, \quad a_2 = \Gamma_L b_2$$

and

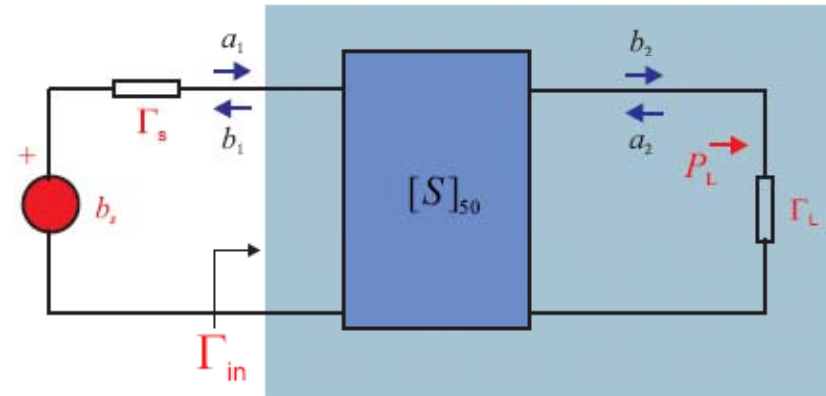
$$b_2 = S_{21}a_1 + S_{22}a_2 = S_{21} \frac{b_s}{1 - \Gamma_{in}\Gamma_s} + S_{22}\Gamma_L b_2$$

Hence

$$b_2 = \frac{S_{21}b_s}{(1 - \Gamma_{in}\Gamma_s)(1 - S_{22}\Gamma_L)}$$

Therefore

$$G_T = \frac{P_L}{P_A} = \frac{|b_2|^2 - |a_2|^2}{\frac{|b_s|^2}{1 - |\Gamma_s|^2}} = \frac{|b_2|^2 (1 - |\Gamma_L|^2)}{\frac{|b_s|^2}{1 - |\Gamma_s|^2}} = \frac{(1 - |\Gamma_s|^2) |S_{21}|^2 (1 - |\Gamma_L|^2)}{|1 - \Gamma_{in}\Gamma_s|^2 |1 - S_{22}\Gamma_L|^2}$$



Available Power Gain

$$G_A \equiv \frac{\text{Available Power from load}}{\text{Available Power from source}} = \frac{P_{L,\max}}{P_A}$$

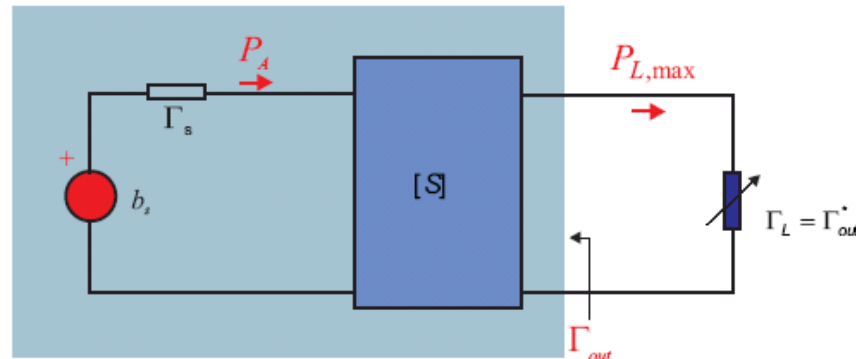
So from the Transducer Power Gain G_T , Available Power Gain when $\Gamma_L = \Gamma_{out}^*$

$$G_A = G_T \Big|_{\Gamma_L = \Gamma_{out}^*} = \frac{(1 - |\Gamma_s|^2) |S_{21}|^2 (1 - |\Gamma_L|^2)}{|1 - \Gamma_{out} \Gamma_L|^2 |1 - S_{11} \Gamma_s|^2} \Big|_{\Gamma_L = \Gamma_{out}^*} = \frac{(1 - |\Gamma_s|^2) |S_{21}|^2}{|1 - S_{11} \Gamma_s|^2 (1 - |\Gamma_{out}|^2)}$$

Therefore

$$G_A = \frac{(1 - |\Gamma_s|^2) |S_{21}|^2}{|1 - S_{11} \Gamma_s|^2 (1 - |\Gamma_{out}|^2)}$$

It represents input mismatch by Γ_s



Available Power Gain Derivation

(Another Derivation of Available Power Gain)

Thevenin Representation at the output side

$$\frac{a_1}{b_s} = \frac{1}{1 - S_{11}\Gamma_s}$$

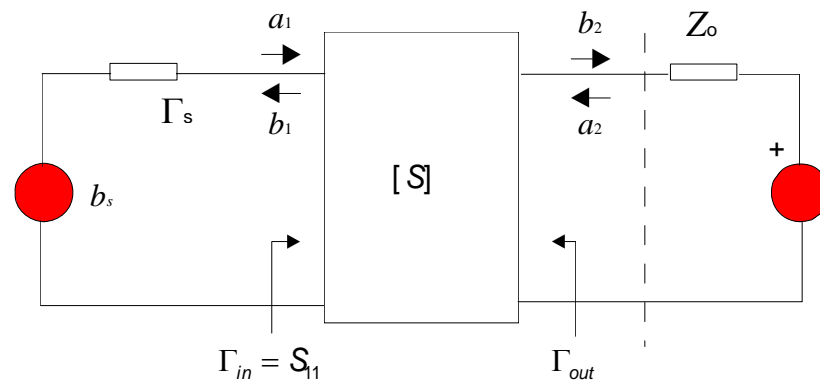
$$b_2 = S_{21}a_1 + \Gamma_{out}a_2 = S_{21}\frac{b_s}{1 - S_{11}\Gamma_s} + \Gamma_{out}a_2 = b_G + S_{22}a_2$$

Hence

$$P_{Ao} = \frac{|b_s|^2 |S_{21}|^2}{|1 - S_{11}\Gamma_s|^2 (1 - |\Gamma_{out}|^2)}, \text{ and } P_{Ai} = \frac{|b_s|^2}{1 - |\Gamma_s|^2}$$

Therefore

$$G_A = \frac{(1 - |\Gamma_s|^2) |S_{21}|^2}{|1 - S_{11}\Gamma_s|^2 (1 - |\Gamma_{out}|^2)}$$



Power Gain

$G_p \equiv \frac{\text{Delivered Power to the Load}}{\text{Input Power at the Device Input}}$
 Input power

$$P_{in} = |a_1|^2 (1 - |\Gamma_{in}|)^2 = \frac{|b_s|^2 (1 - |\Gamma_{in}|)^2}{|1 - \Gamma_{in} \Gamma_s|^2}$$

$$P_L = \frac{|b_s|^2 |S_{21}|^2 (1 - |\Gamma_L|^2)}{|1 - \Gamma_{in} \Gamma_s|^2 |1 - S_{22} \Gamma_L|^2}$$

Hence

$$G_p = \frac{|S_{21}|^2 (1 - |\Gamma_L|^2)}{(1 - |\Gamma_{in}|)^2 |1 - S_{22} \Gamma_L|^2}$$

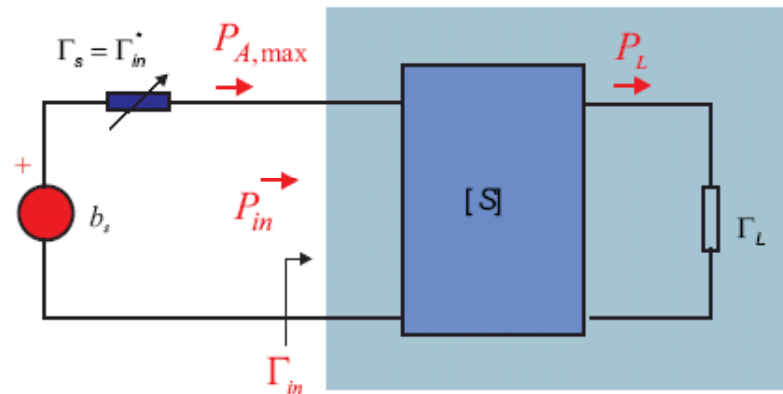
This is the Power gain from Transducer Power gain when $\Gamma_s = \Gamma_{in}^*$

$$G_p = G_T \Big|_{\Gamma_s = \Gamma_{in}^*} = \frac{(1 - |\Gamma_s|^2) |S_{21}|^2 (1 - |\Gamma_L|^2)}{|1 - \Gamma_{in} \Gamma_s|^2 |1 - S_{22} \Gamma_L|^2} \Big|_{\Gamma_s = \Gamma_{in}^*} = \frac{|S_{21}|^2 (1 - |\Gamma_L|^2)}{(1 - |\Gamma_{in}|)^2 |1 - S_{22} \Gamma_L|^2}$$

Therefore

$$G_p = \frac{|S_{21}|^2 (1 - |\Gamma_L|^2)}{(1 - |\Gamma_{in}|)^2 |1 - S_{22} \Gamma_L|^2}$$

It represent the output mismatch by Γ_L



Unilateral Gains

In Unilateral Case , ie $S_{12} = 0$, Transducer Power Gain becomes

$$G_{TU} = G_T \Big|_{S_{12}=0} = \frac{(1-|\Gamma_s|^2)|S_{21}|^2(1-|\Gamma_L|^2)}{|1-S_{11}\Gamma_s|^2|1-\Gamma_{out}\Gamma_L|^2} \Big|_{S_{12}=0} = \frac{(1-|\Gamma_s|^2)|S_{21}|^2(1-|\Gamma_L|^2)}{|1-S_{11}\Gamma_s|^2|1-S_{22}\Gamma_L|^2}$$

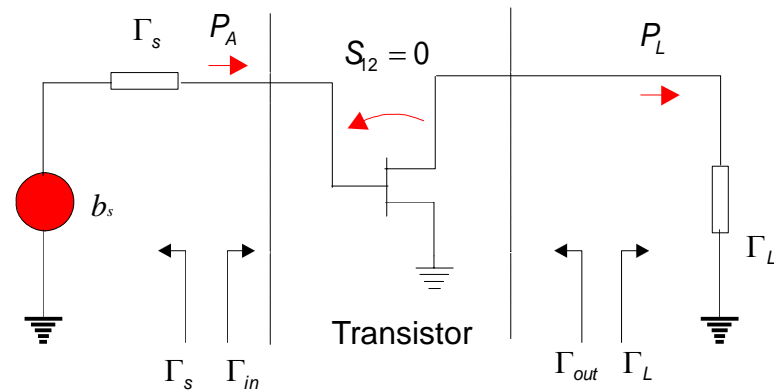
When conjugate match both the input and output

$$G_{TU,max} = \frac{|S_{21}|^2}{(1-|S_{11}|^2)(1-|S_{22}|^2)}$$

In general, $S_{12} \neq 0$, $G_{TU,max}$ becomes Mason's Gain U

$$U = \frac{\left| \frac{S_{21}}{S_{12}} - 1 \right|^2}{2k \left| \frac{S_{21}}{S_{12}} \right| - 2 \operatorname{Re} \left(\frac{S_{21}}{S_{12}} \right)}$$

This is used as the device Passivity and Activity



Load Stability Circle

Q; Input impedance \rightarrow negative for some $\Gamma_L \rightarrow |\Gamma_{in}| > 1$

The region of Γ_L ? $\rightarrow |\Gamma_{in}| < 1$

Load Stability Circle;

the locus of Γ_L which makes $|\Gamma_{in}| = 1$

From
$$\Gamma_{in} = S_{11} + \frac{S_{12}S_{21}\Gamma_L}{1 - S_{22}\Gamma_L}$$

$$\Gamma_L = \frac{\Gamma_{in} - S_{11}}{S_{22}\Gamma_{in} - D} = \frac{1}{S_{22}} + \frac{D/S_{22} - S_{11}}{S_{22}\Gamma_{in} - D}$$

where

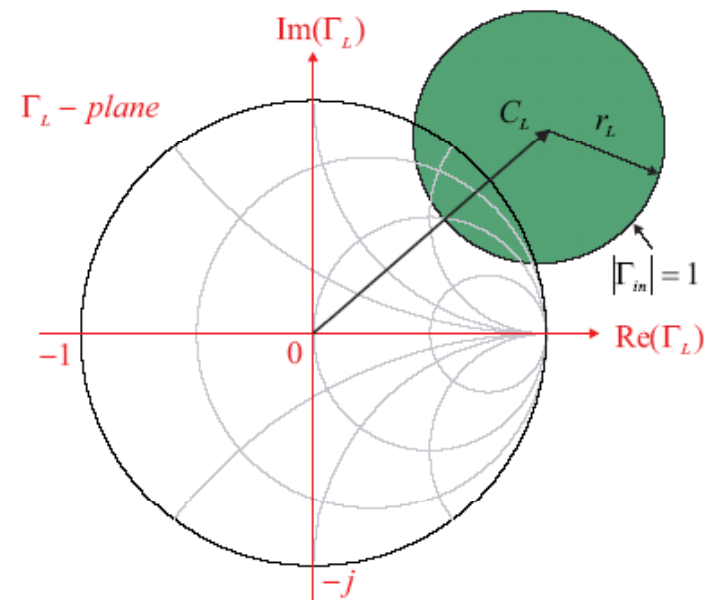
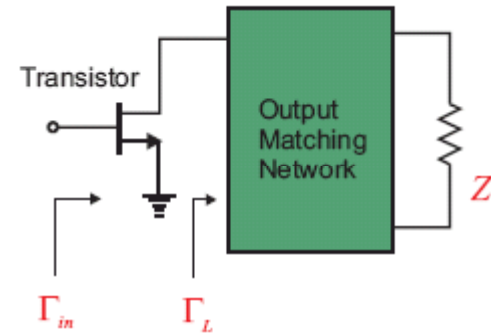
$$D = S_{11}S_{22} - S_{21}S_{12}$$

Center C_L and Radius r_L

$$C_L = \frac{S_{22}^* - D^*S_{11}}{|S_{22}|^2 - |D|^2}, \text{ and } r_L = \frac{|S_{12}S_{21}|}{||S_{22}|^2 - |D|^2|}$$

Stable Region

$$\left\{ \begin{array}{l} |S_{11}| < 1 \\ \text{Outside the Circle} \end{array} \right. \text{ or } \left\{ \begin{array}{l} |S_{11}| > 1 \\ \text{Inside the Circle} \end{array} \right.$$



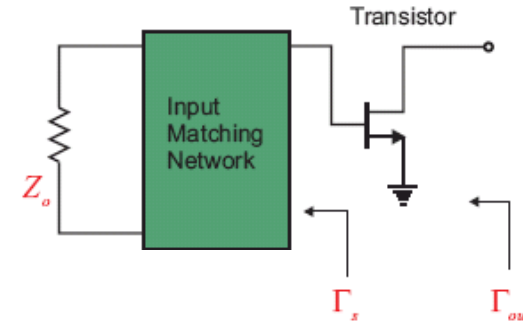
Source Stability Circle

Q; Output impedance \rightarrow negative for some $\Gamma_s \rightarrow |\Gamma_{out}| > 1$

The region of Γ_s ? $\rightarrow |\Gamma_{out}| < 1$

Source Stability Circle;

the locus of Γ_s which makes $|\Gamma_{out}| = 1$



From
$$\Gamma_{out} = S_{22} + \frac{S_{12}S_{21}\Gamma_s}{1 - S_{22}\Gamma_s}$$

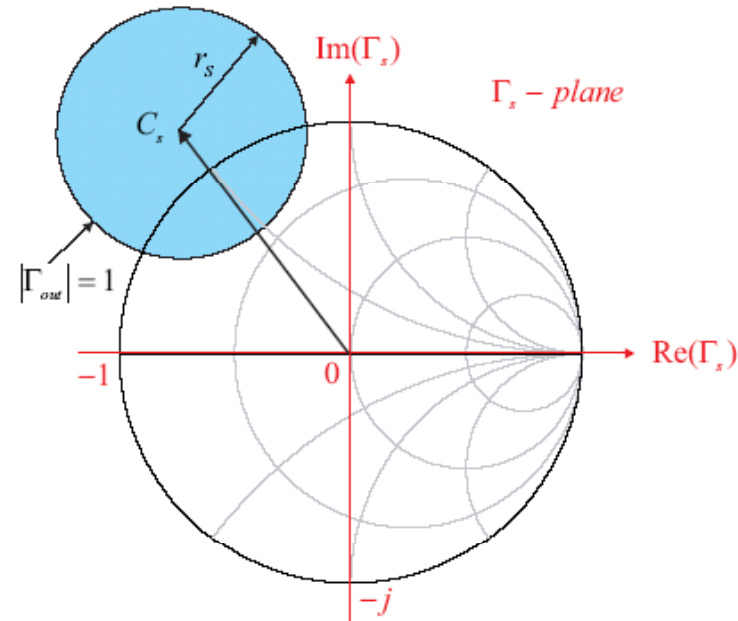
Changing 1 \rightarrow 2; Source stability circle

Center C_s and Radius r_s

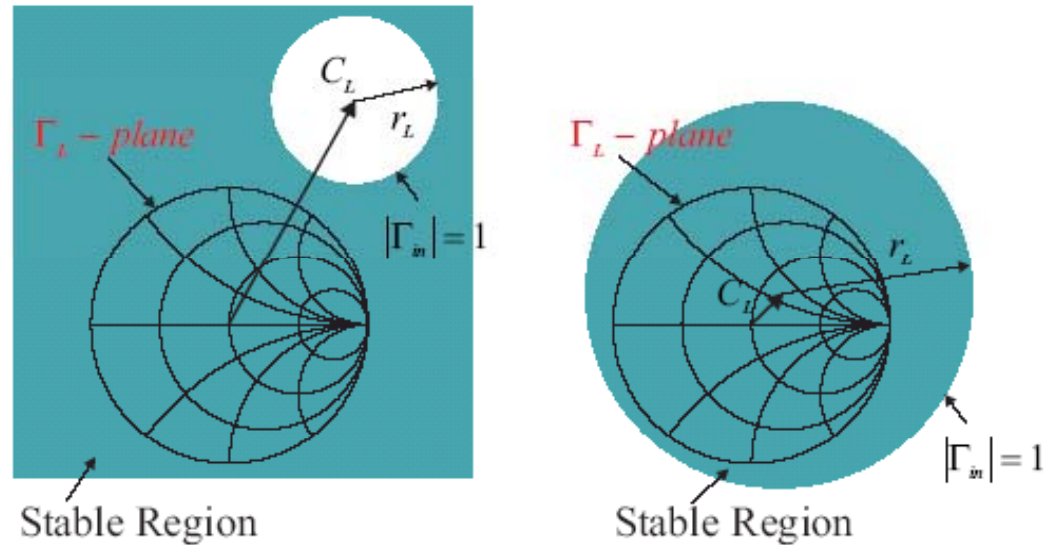
$$C_s = \frac{S_{11}^* - D^* S_{22}}{|S_{11}|^2 - |D|^2}, \text{ and } r_s = \frac{|S_{12}S_{21}|}{||S_{11}|^2 - |D|^2|}$$

Stable Region

$$\begin{cases} |S_{22}| < 1 \\ \text{Outside the Circle} \end{cases} \text{ or } \begin{cases} |S_{22}| > 1 \\ \text{Inside the Circle} \end{cases}$$



Unconditional Stability Condition



The Necessary and Sufficient Condition for Stability

$$|S_{11}| < 1, \begin{cases} |C_L| - r_L > 1 \\ \text{or} \\ r_L - |C_L| > 1 \end{cases}, \text{ and } |S_{22}| < 1, \begin{cases} |C_s| - r_s > 1 \\ \text{or} \\ r_s - |C_s| > 1 \end{cases}$$

Then

$$\text{Stability Factor; } k = \frac{1 - |S_{22}|^2 - |S_{11}|^2 + |D|^2}{2|S_{12}S_{21}|} > 1, \quad |S_{12}S_{21}| < 1 - |S_{22}|^2, \text{ and } |S_{12}S_{21}| < 1 - |S_{11}|^2$$

Conjugate Match & MAG (Maximum Available Gain)

In Stable Case, Maximum Gain occurs at Simultaneous Conjugate Match

$$\Gamma_s^* = S_{11} + \frac{S_{12}S_{21}\Gamma_L}{1 - S_{22}\Gamma_L}, \quad \Gamma_L^* = S_{22} + \frac{S_{12}S_{21}\Gamma_s}{1 - S_{11}\Gamma_s}$$

These are Simultaneous Equations and Meaningful solutions

$$\Gamma_{sM} = \frac{C_1^*}{|C_1|} \left[\frac{B_1}{2|C_1|} - \sqrt{\frac{B_1^2}{4|C_1|^2} - 1} \right], \quad \text{and} \quad \Gamma_{LM} = \frac{C_2^*}{|C_2|} \left[\frac{B_2}{2|C_2|} - \sqrt{\frac{B_2^2}{4|C_2|^2} - 1} \right]$$

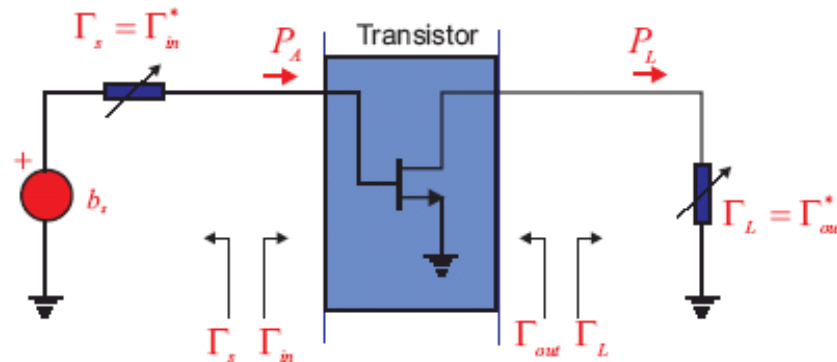
where

$$C_1 = S_{11} - DS_{22}^*, \quad C_2 = S_{22} - DS_{11}^*, \quad B_1 = 1 - |S_{22}|^2 + |S_{11}|^2 - |D|^2, \quad B_2 = 1 - |S_{11}|^2 + |S_{22}|^2 - |D|^2$$

$$D = S_{11}S_{22} - S_{12}S_{21}$$

and the gain becomes maximum

$$MAG = G_{\max} = \left| \frac{S_{21}}{S_{12}} \right| (k - \sqrt{k^2 - 1})$$



MSG (Maximum Stable Gain)

For stable condition

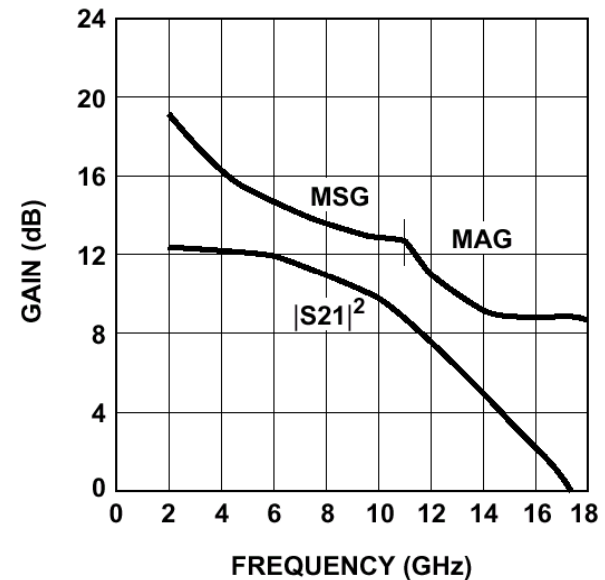
$$MAG(\text{Maximum Available Gain}) = G_{\max} = \left| \frac{S_{21}}{S_{12}} \right| (k - \sqrt{k^2 - 1})$$

When the device is **unstable**, the power Gain attainable is equal to

$$MSG(\text{Maximum Stable Gain}) = MAG|_{k=1} = \left| \frac{S_{21}}{S_{12}} \right|$$

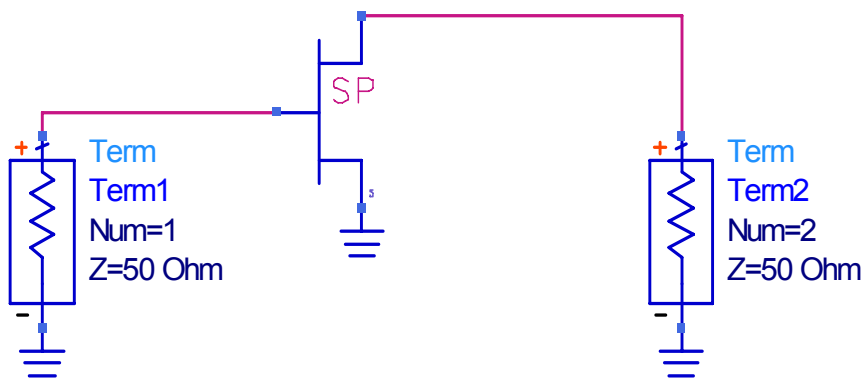
Note

1. MSG and MAG is Power Gains
2. MSG or MAG are maximum Power Gain obtainable from devices
3. MSG can be obtained when $k = 1$



ADS Example(Ex 8.3)

sp_fuj_FHX35LG_19920501
SNP1
Bias="Fet: Vds=3V Ids=10mA"
Frequency="{0.10 - 18.00} GHz"



S_Param
SP1
Start=1.0 GHz
Stop=18.0 GHz
Step=1.0 GHz

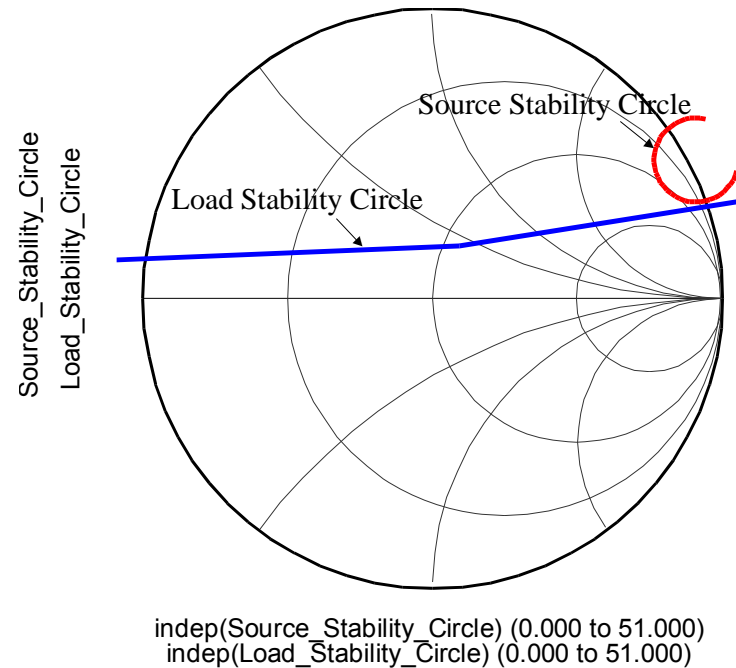
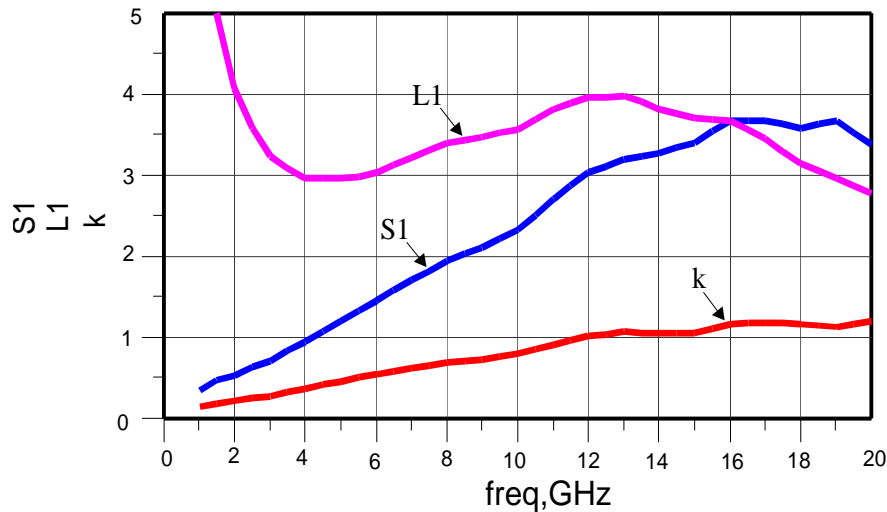
Plot $\frac{1-|S_{11}|^2}{|S_{12}S_{21}|}$, $\frac{1-|S_{22}|^2}{|S_{12}S_{21}|}$, and k . Draw Load and Source stability circles at 1GHz

ADS Example(Ex 8.3)

$K = \text{stab_fact}(S)$, $S1 = (1 - \text{mag}(S11))^{**2} / \text{mag}(S12 * S21)$, $S1 = (1 - \text{mag}(S11))^{**2} / \text{mag}(S12 * S21)$

Source_stability_Circle = s_stab_circle(S[0],51)

Load_stability_Circle = l_stab_circle(S[0],51)



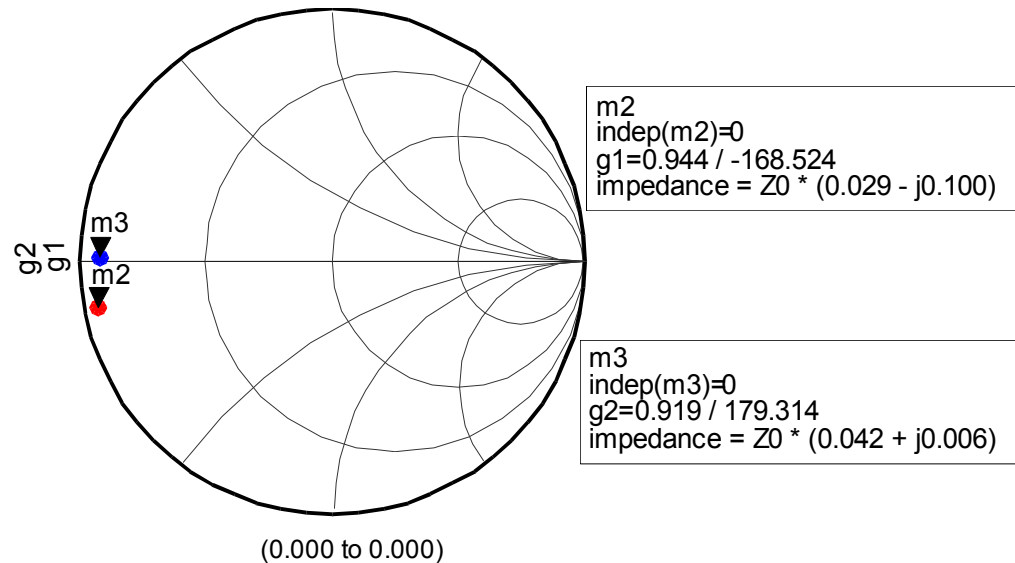
ADS Example(Ex 8.4)

For the S-parameters in ex8-2 Plot max gain and conjugate matching points at 12 GHz

Eqn m=find_index(freq, 12G)

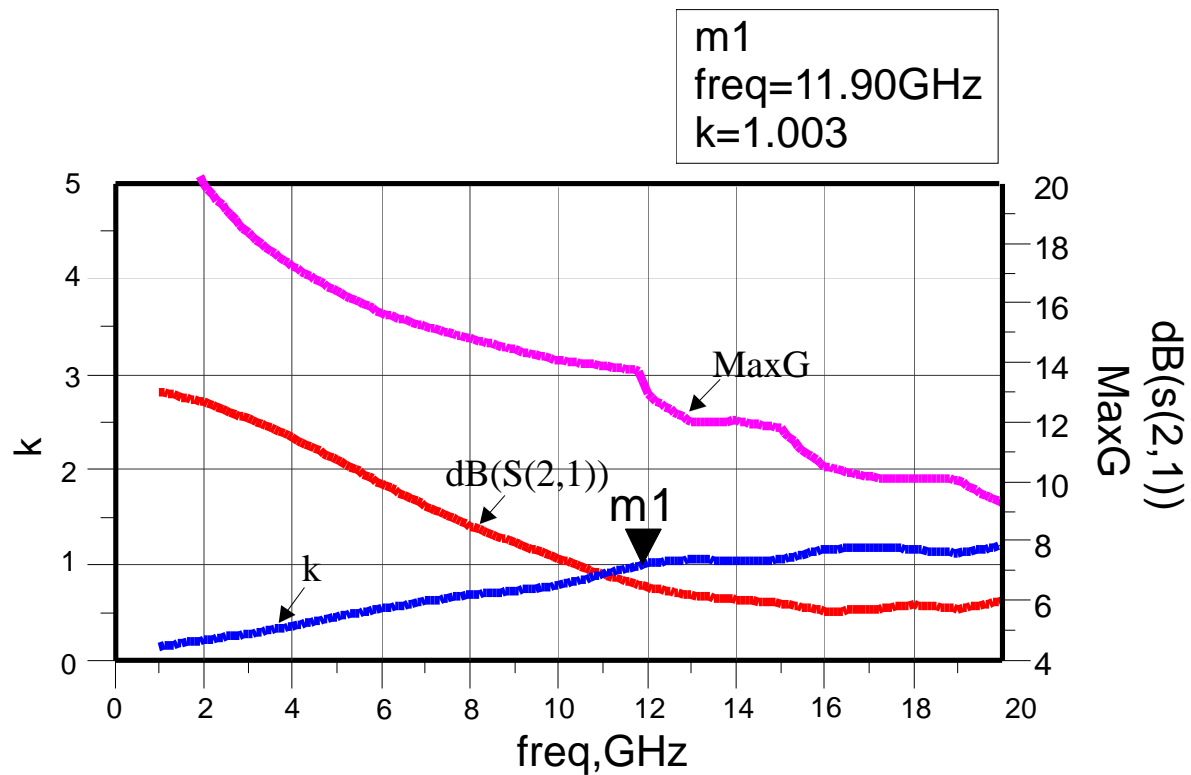
Eqn g1=sm_gamma1(S[m])

Eqn g2=sm_gamma2(S[m])



ADS Example(Ex 8.4)

Eqn MaxG=max_gain(S)



Available Gain Circles

Q: Choice of Γ_s → Not Γ_{sM} → But $\Gamma_L = (\Gamma_{out})^*$ → Gain Degradation?
→ The locus of Γ_s giving the equal gain degradation

From Available Gain

$$G_A = \frac{(1 - |\Gamma_s|^2) |S_{21}|^2}{|1 - S_{11}\Gamma_s|^2 (1 - |\Gamma_{out}|^2)} = g_a |S_{21}|^2$$

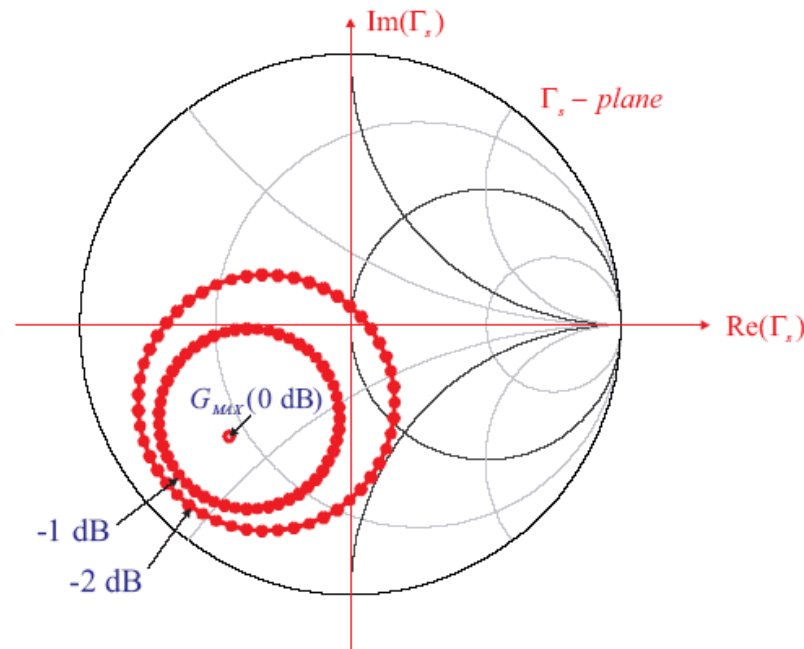
where g_a normalized gain by $|S_{21}|^2$ and

$$g_{a,max} = \frac{k - \sqrt{k^2 - 1}}{|S_{21}S_{12}|}$$

The Center and Radius

$$C_A = \frac{g_a (S_{11}^* - D^* S_{22})}{1 + g_a (|S_{11}|^2 - |D|^2)}$$

$$r_A = \frac{\sqrt{1 - 2k |S_{12}S_{21}| g_a + |S_{12}S_{21}|^2 g_a^2}}{|1 + g_a (|S_{11}|^2 - |D|^2)|}$$



Note: It represents the gain degradation by the Source Mismatch

Power Gain Circles

Q: Choice of $\Gamma_L \rightarrow$ Not $\Gamma_{LM} \rightarrow$ But $\Gamma_s = (\Gamma_{in})^* \rightarrow$ Gain Degradation?
 \rightarrow The locus of Γ_L giving the equal gain degradation

From Power Gain

$$G_p = \frac{(1 - |\Gamma_L|^2) |S_{21}|^2}{|1 - S_{22}\Gamma_L|^2 (1 - |\Gamma_{in}|^2)} = g_p |S_{21}|^2$$

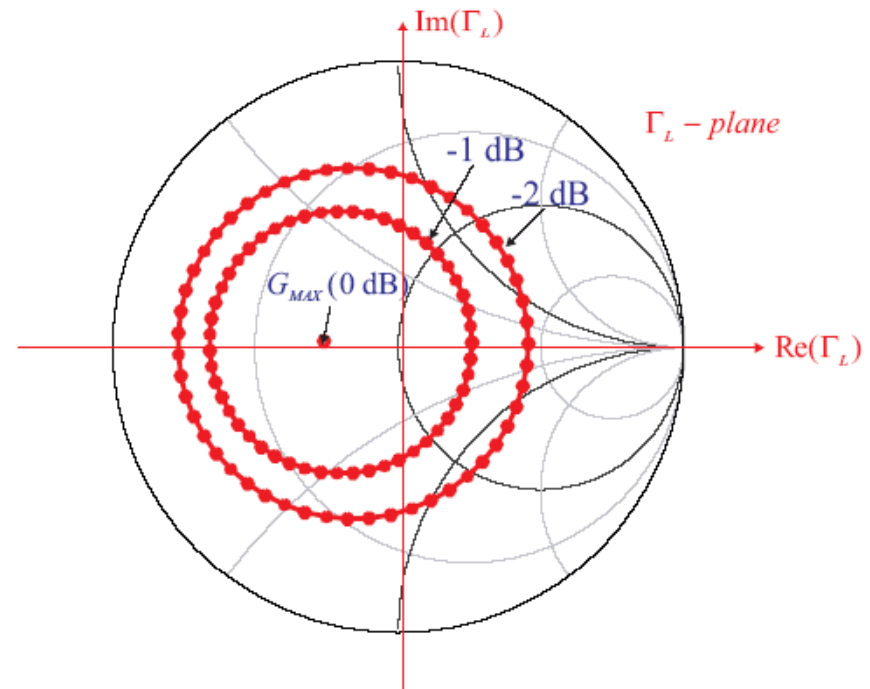
as g_a , where g_p normalized gain by $|S_{21}|^2$ and

$$g_{p,max} = \frac{k - \sqrt{k^2 - 1}}{|S_{21}S_{12}|}$$

The Center and Radius by changing C_A, r_A 1 \rightarrow 2

$$C_p = \frac{g_p (S_{22}^* - D^* S_{11})}{1 + g_p (|S_{22}|^2 - |D|^2)}$$

$$r_p = \frac{\sqrt{1 - 2k |S_{12}S_{21}| g_p + |S_{12}S_{21}|^2 g_p^2}}{|1 + g_p (|S_{22}|^2 - |D|^2)|}$$



Note: It represents the gain degradation by the Load Mismatch

Noise Figure & Noise Parameters

Ref) G. Gonzalez, *Microwave Transistor Amplifier*, 2nd Edition, Prentice Hall

$$F = \frac{\text{Actual noise power output}}{\text{Power output due to source noise}} = \frac{P_{no}}{P_{ns}}$$

$$= F_{\min} + \frac{R_n}{G_s} |Y_s - Y_{opt}|^2$$

where

$$Y_s = G_s + jB_s, \quad \text{and} \quad Y_{opt} = G_{opt} + jB_{opt}$$

Noise parameters

$R_n(\omega)$; Noise resistance

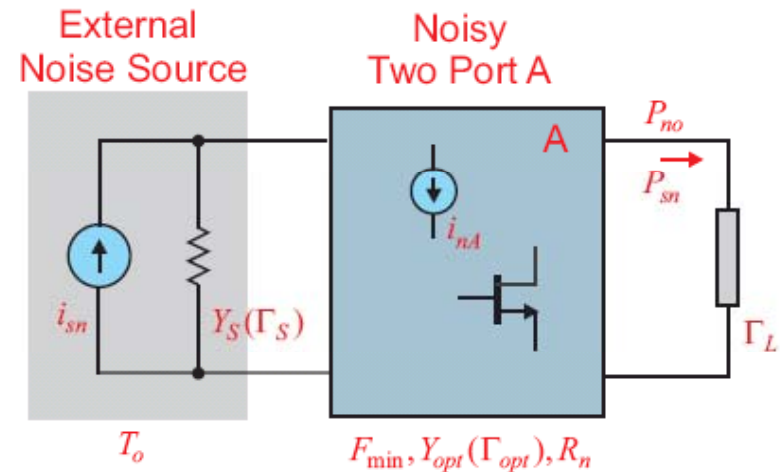
$Y_{opt}(\omega)$; Optimum Source Admittance

F_{\min} ; Min. Noise Figure

Rearranging

$$F = F_{\min} + \frac{R_n}{G_s} |Y_s - Y_{opt}|^2 = F_{\min} + \frac{4r_n |\Gamma_s - \Gamma_{opt}|^2}{(1 - |\Gamma_s|^2) |1 + \Gamma_{opt}|^2}$$

$$\leftarrow Y_s = \frac{1}{Z_o} \cdot \frac{1 - \Gamma_s}{1 + \Gamma_s}, \quad Y_{opt} = \frac{1}{Z_o} \cdot \frac{1 - \Gamma_{opt}}{1 + \Gamma_{opt}}, \quad \text{and} \quad r_n = \frac{R_n}{Z_o}$$



Noise Circles

**Q; Choice of $\Gamma_s \rightarrow$ Not $\Gamma_{opt} \rightarrow$ NF degradation from F_{min}
 \rightarrow The locus of Γ_s giving the equal degradation of NF**

From

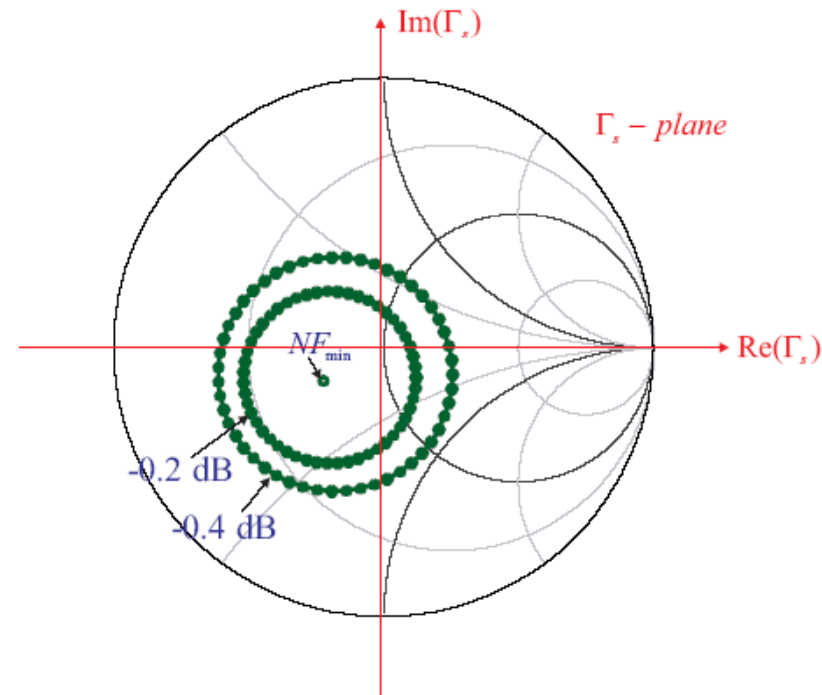
$$F = F_{min} + \frac{4r_n |\Gamma_s - \Gamma_{opt}|^2}{(1 - |\Gamma_s|^2)(1 + |\Gamma_{opt}|^2)}$$

$$\rightarrow \frac{|\Gamma_s - \Gamma_{opt}|^2}{(1 - |\Gamma_s|^2)} = \frac{F - F_{min}}{4r_n} |1 + \Gamma_{opt}|^2 = N_i$$

The center & Radius

$$C_F = \frac{\Gamma_{opt}}{1 + N_i},$$

$$R_F = \frac{1}{N_i + 1} \sqrt{N_i^2 + N_i(1 - |\Gamma_{opt}|^2)}$$



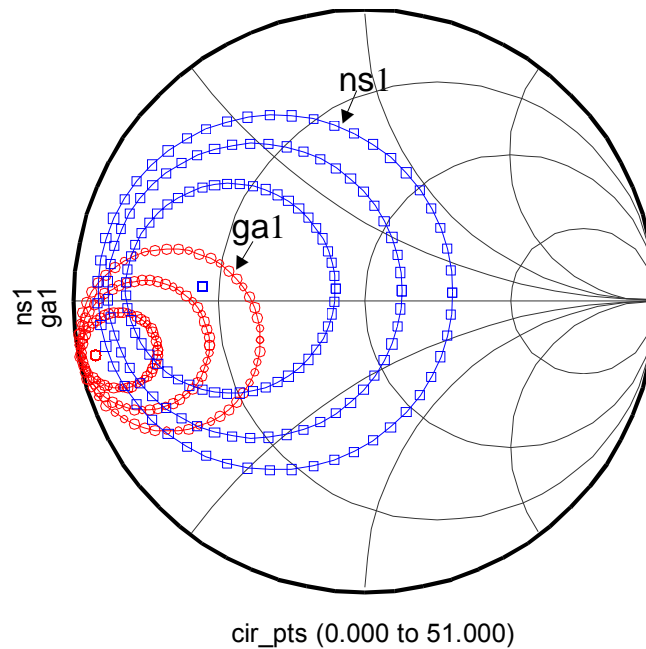
It represents the NF degradation caused by Γ_s

ADS Example(Ex 8.4)

For the S-parameters in ex8-2 Plot Available gain and noise circles at 12 GHz

Eqn ga1=ga_circle(S[m])

Eqn ns1=ns_circle({0, 1,2,3}+NFmin[m], NFmin[m], Sopt[m], Rn[m]/50, 51)



Gain Summary

Transducer power gain in 50-ohm	$G_T = S_{21} ^2$
Transducer power Gain	$G_T = \frac{(1 - \Gamma_L ^2) S_{21} ^2 (1 - \Gamma_s ^2)}{ (1 - S_{22} \Gamma_L)(1 - S_{11} \Gamma_s) - S_{12} S_{21} \Gamma_L \Gamma_s ^2}$
Power Gain(Output mismatch)	$G_p = \frac{(1 - \Gamma_L ^2) S_{21} ^2}{ 1 - S_{22} \Gamma_L ^2 (1 - \Gamma_{in} ^2)}$
Available power Gain (Input Mismatch)	$G_A = \frac{P_{Ao}}{P_{Ai}} = \frac{(1 - \Gamma_s ^2) S_{21} ^2}{ 1 - S_{11} \Gamma_s ^2 (1 - \Gamma_{out} ^2)}$
Mason's unilateral Gain (Passivity and Activity)	$U = \frac{ S_{21}/S_{12} - 1 ^2}{2k S_{21}/S_{12} - 2 \operatorname{Re}(S_{21}/S_{12})}$
Maximum Unilateral Power gain (Approximation)	$G_{TUmax} = \frac{ S_{21} ^2}{(1 - S_{22} ^2)(1 - S_{11} ^2)}$
Maximum Stable gain	$G_{ms} = \left \frac{S_{21}}{S_{12}} \right $
Maximum Available gain	$G_{ma} = \left \frac{S_{21}}{S_{12}} \right (k - \sqrt{k^2 - 1})$

Circles Summary

Circles	Center	Radius
Available Gain	$C_A = \frac{g_a(S_{11}^* - D^* S_{22})}{1 + g_a(S_{11} ^2 - D ^2)}$	$r_A = \frac{\sqrt{1 - 2k S_{12}S_{21} g_a + S_{12}S_{21} ^2 g_a^2}}{ 1 + g_a(S_{11} ^2 - D ^2) }$
Power Gain	$C_p = \frac{g_p(S_{22}^* - D^* S_{11})}{1 + g_p(S_{22} ^2 - D ^2)}$	$r_p = \frac{\sqrt{1 - 2k S_{12}S_{21} g_p + S_{12}S_{21} ^2 g_p^2}}{ 1 + g_p(S_{22} ^2 - D ^2) }$
Noise Mismatch	$C_F = \frac{\Gamma_{opt}}{1 + N_i}$	$r_F = \frac{\sqrt{N_i^2 + N_i(1 - \Gamma_{opt} ^2)}}{(1 + N_i)}$
Input Stability	$C_s = \frac{S_{11}^* - D^* S_{22}}{ S_{11} ^2 - D ^2}$	$r_s = \frac{ S_{12}S_{21} }{ S_{11} ^2 - D ^2 }$
Output Stability	$C_L = \frac{S_{22}^* - D^* S_{11}}{ S_{22} ^2 - D ^2}$	$r_L = \frac{ S_{12}S_{21} }{ S_{22} ^2 - D ^2 }$

$$g_a = \frac{G_A}{|S_{21}|^2}, \quad g_p = \frac{G_p}{|S_{21}|^2}, \quad \text{and} \quad k = \frac{1 - |S_{11}|^2 - |S_{22}|^2 + |D|^2}{2|S_{12}S_{21}|}$$

Summary & Reference

- Gains → Transducer Power Gain, Available Power Gain, Power Gain

Circles → Available Gain Circle, Power Gain Circle

Load and Source Stability Circles

- NF, Noise Circles

- Reference

G. Gonzalez, *Microwave Transistor Amplifier*, 2nd Edition, Prentice Hall

Lab 7-2

- 2SC4226의 S-parameter를 측정하고(100MHz-1GHz) Stability를 확인한 후 이의 Input, output conjugate Matching impedance 및 MSG, MAG를 plot 하시오

Lab 7-3

- 2SC4226의 Available Gain Circle 및 noise Circle Stability circle을 그리시오

Lab 7-4

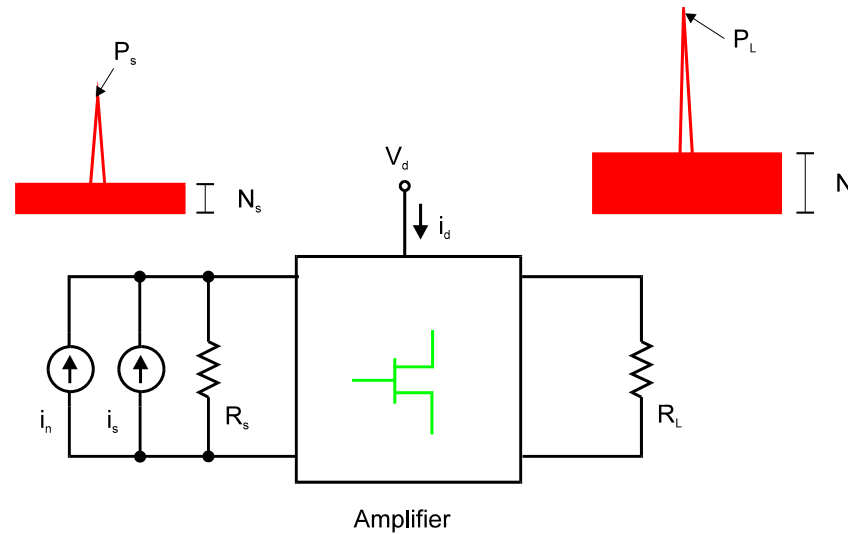
- 강의에 보인 2SC4226의 5V, 30mA bias를 15V 전원을 이용한 active bias 회로를 설계하고 이를 ADS를 이용 확인하시오.
- (pnp TR 2N2907)

LNA Design

Contents

- Problem
- Design with ADS
 - Lumped Design
 - Distributed Design Examples
- Conclusion and reference

Amplifier in Noisy Environment



Problem

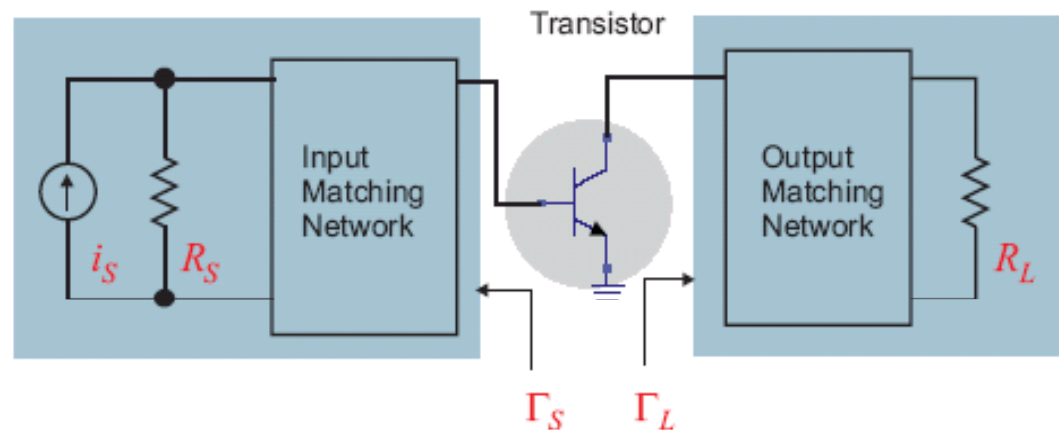
--Minimize the Noise figure

$$\text{Noise Factor: } F' = \frac{P_L / N_L}{P_s / N_s} = 1 + \frac{N_a}{N_s G}$$

$$\text{Noise Figure: } F = 10 \log(F') \text{ [dB]}$$

Design Procedure for LNA

- Active Device Selection ← (**MAG > G**, **Nfmin < NF** @ Design frequency)
- Pick up the optimum Source & Load impedances (**Circles Drawing**)
- Matching network design
- Bias Circuit Adding
- Stability Check @ other frequencies
- Layout



Lumped LNA Example

Active Device Preparation for ADS

- S-parameter inputting to ADS
 - S-parameter Check
 - Minimum Noise Check
- Large Signal Model
 - Curve Tracing
 - S-parameter Check
 - Minimum Noise Check

S-Parameter Inputting

TouchStone Format

```
! NEC710
# GHZ S MA R 50
2 .95 -26 3.57 157 .04 76 .66 -14
22 .60 -144 1.30 40 .14 40 .56 -85
! NOISE PARAMETERS
4 .7 .64 69 .38
18 2.7 .46 -33 .40
(Freq NFmin Sopt Rn/Zo)
```

CitiFile Format

```
CITIFILE A.01.00
#NA VERSION HP8510B.05.00
NAME MEMORY
#NA REGISTER 1
VAR FREQ MAG 5
DATA S RI
    VAR_LIST_BEGIN
    1000000000
    2000000000
    2500000000
    3000000000
    VAR_LIST_END
BEGIN
-1.31189E-3,-1.47980E-3
-3.67867E-3,-0.67782E-3
-3.43990E-3,0.58746E-3
-2.70664E-4,-9.76175E-4
0.65892E-4,-9.61571E-4
END
```

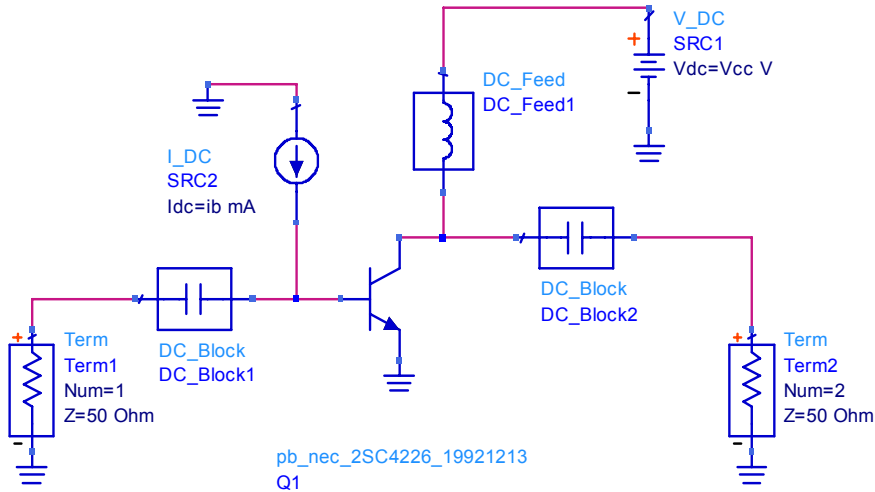
- Text-editor

File
Editing

From
Network
Analyzer



Large Signal Model Check

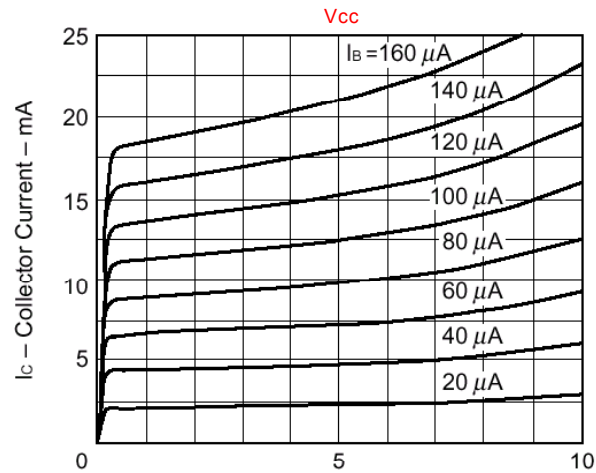
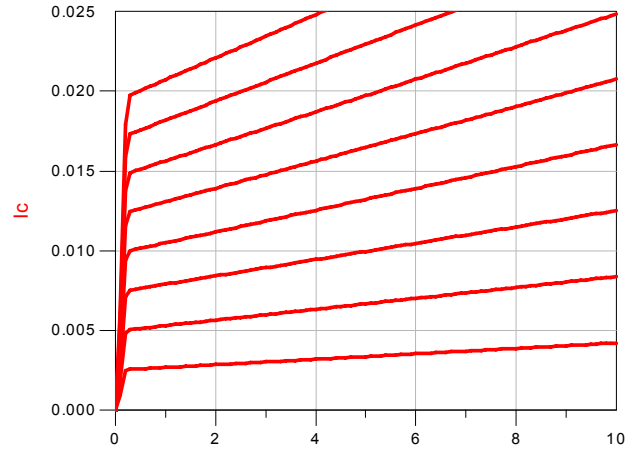


pb_nec_2SC4226_19921213
Q1

Var Eqn
VAR
VAR1
Vcc=1.0
ib=1.0

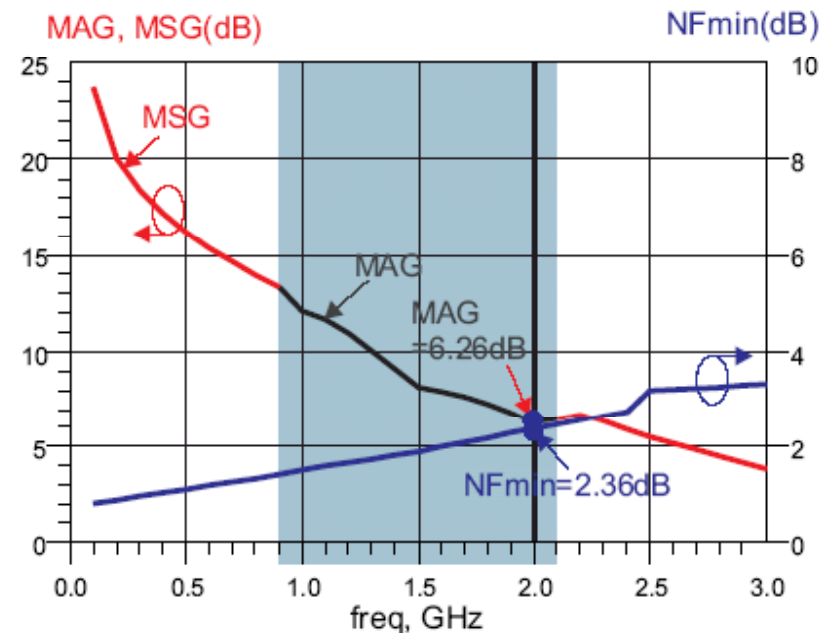
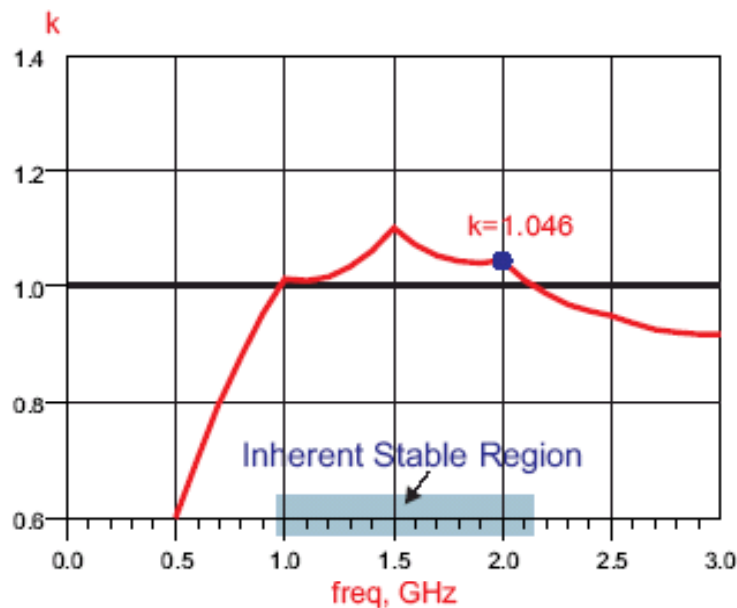
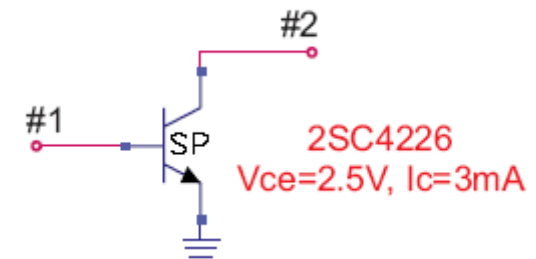
DC
DC1
SweepVar="Vcc"
Start=0
Stop=10

PARAMETER SWEEP
ParamSweep
ib
SweepVar="ib"
SimInstanceName[1]="DC1"
SimInstanceName[2]=
SimInstanceName[3]=
SimInstanceName[4]=
SimInstanceName[5]=
SimInstanceName[6]=
Start=0.02
Stop=0.16

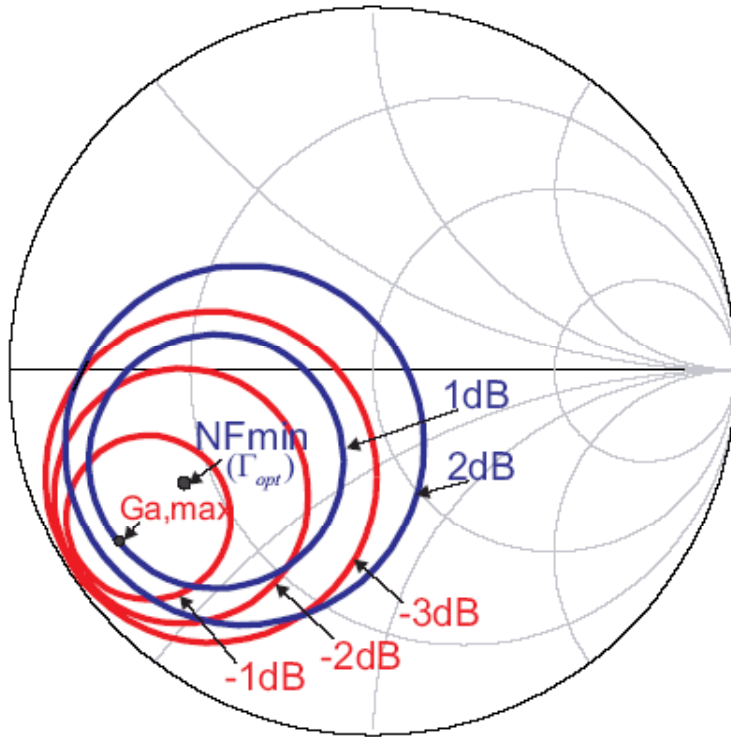


Device for LNA (@ 2GHz, $G > 5\text{dB}$, $NF < 3\text{dB}$)

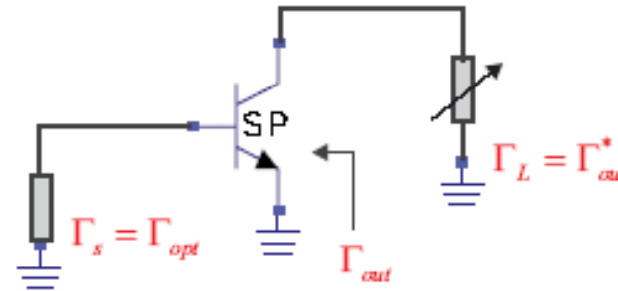
- BJT: 2SC4226 (2.5V, $I_c=3\text{mA}$),
- Stable at 2GHz
- Gain: 6.26dB max, NF; 2.36dB min
- Unstable case \rightarrow Stabilizing



Input & Output Impedance Selection



Ga Circles & Noise Circles @2GHz



Impedance for Min NF

$$\Gamma_s = \Gamma_{opt} = 0.6 \angle 150^\circ$$

$$Z_s = Z_{opt} = 13.4 - j13.0$$

Associated Gain $G_A > 5.26$ dB

Associated Output Impedance

$$\Gamma_L = \Gamma_{out}^* = \left(S_{22} + \frac{S_{12}S_{21}\Gamma_{opt}}{1 - S_{11}\Gamma_{opt}} \right)^*$$

$$= 0.661 \angle 67.7^\circ$$

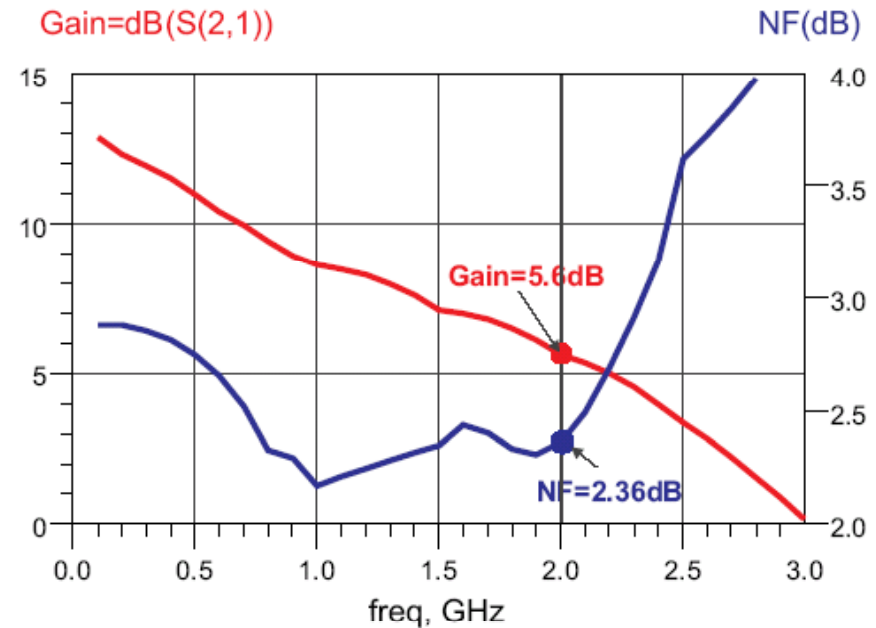
$$Z_L = 30.1 + j65.3$$

Selected Impedance Verification

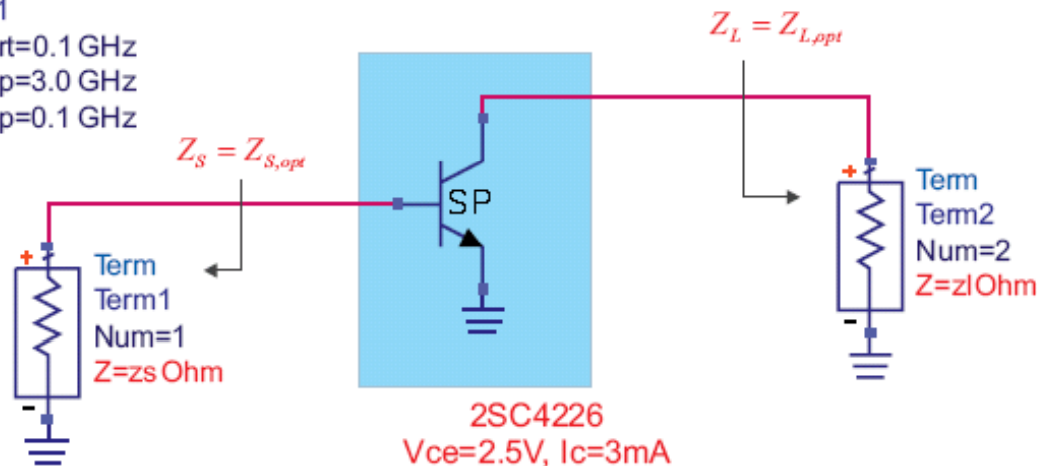
$$Z_S = Z_{S,opt} = 13.4 - j13.0,$$

$$Z_L = Z_{L,opt} = 30.1 + j65.3$$

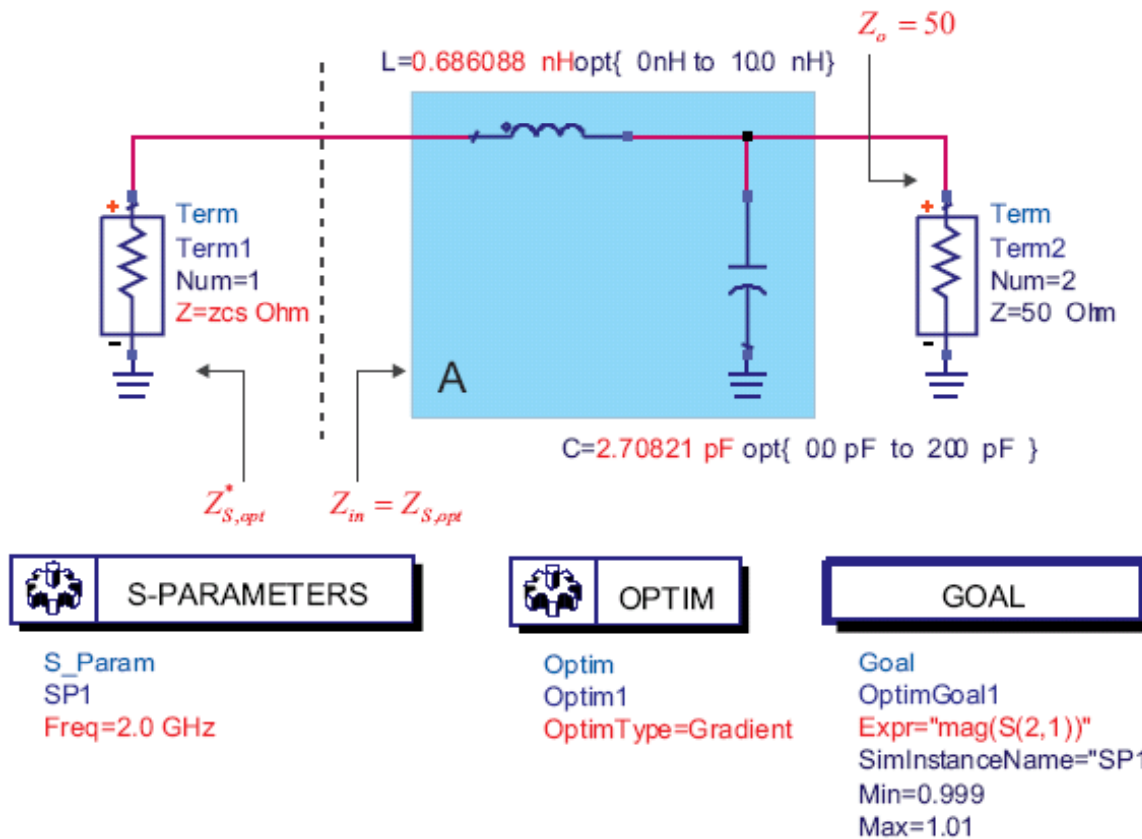
Gain=5.6dB > 5.26dB, NF=2.36dB



S_Param
SP1
Start=0.1 GHz
Stop=3.0 GHz
Step=0.1 GHz



Matching Network Design (Input)



Matching Network by C, L (Ch. 5)

$$Z_o = 50$$

$$\rightarrow Z_{in} = Z_{S,opt} = 13.4 - j13.0$$

Set port #1 Impedance

$$Z_1 = Z_{S,opt}^*$$

Maximum Power Transfer

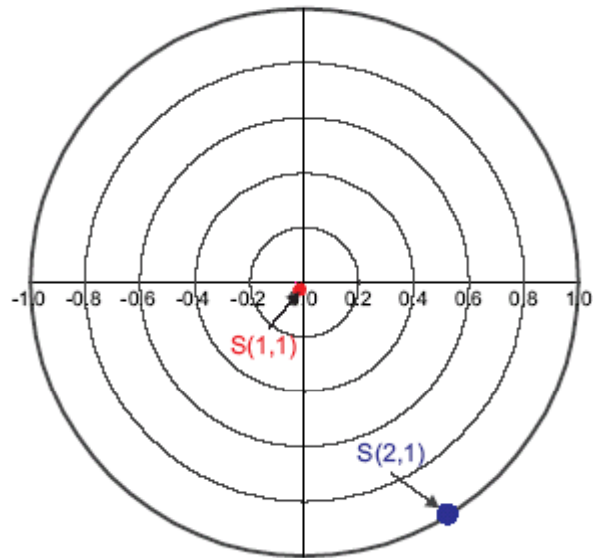
$$Z_{in} = Z_{S,opt}$$

Matching Network Results (Input)

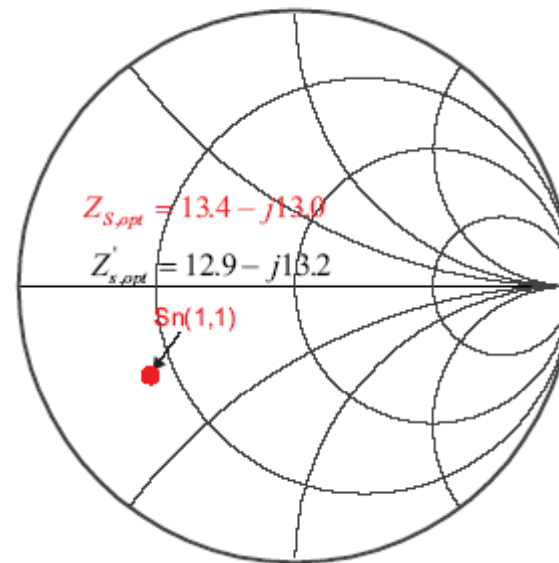
For the S-parameters ($Z_{S,opt}^*$, 50) **at matched condition**

$$S_{11} = \frac{Z_{in} - (Z_{S,opt}^*)^*}{Z_{in} + (Z_{S,opt}^*)^*} = \frac{Z_{in} - Z_{S,opt}}{Z_{in} + Z_{S,opt}} = 0 \quad (\because Z_{in} = Z_{S,opt} \text{ for match}) \text{ and } |S_{21}|^2 = \frac{|b_2|^2}{|a_1|^2} = 1 \rightarrow |S_{21}| = 1$$

$\rightarrow |a_1|^2$; delivered power to 2-port input and **2-port lossless**

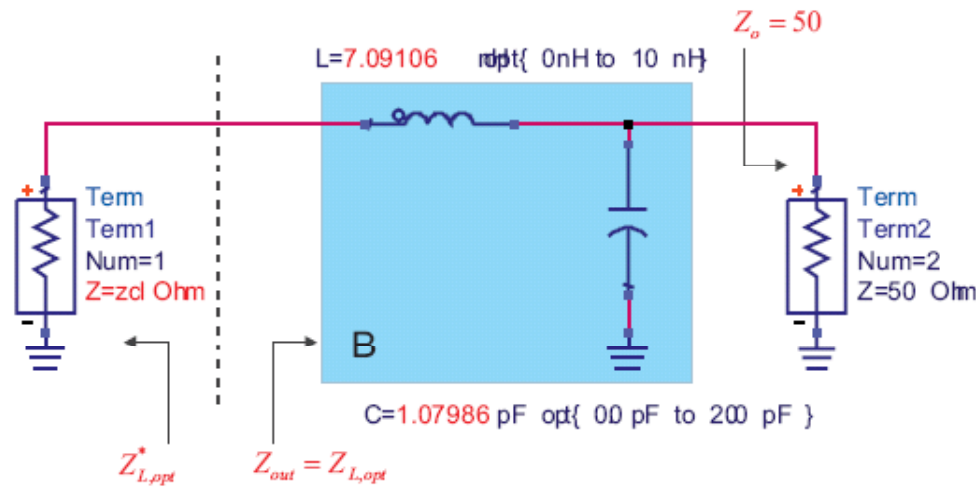


Reference ($Z_{S,opt}^*$, 50)



Reference (50, 50)

Matching Network Design(Output)



Matching Network by C, L

$$Z_o = 50$$

$$\rightarrow Z_{out} = Z_{L,opt} = 30.1 + j65.3$$

Set port #1 Impedance

$$Z_1 = Z_{L,opt}^*$$

Maximum Power Transfer

$$Z_{out} = Z_{L,opt}$$

S-PARAMETERS

S_Param
SP1
Freq=2.0 GHz

OPTIM

Optim
Optim1
OptimType=Gradient

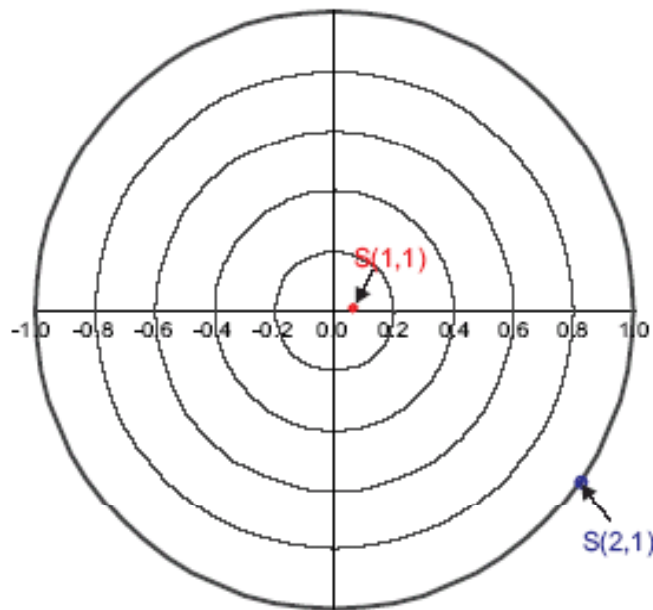
GOAL

Goal
OptimGoal1
Expr='mag(S(2,1))'
SimInstanceName='SP1'
Min=0.999
Max=1.01

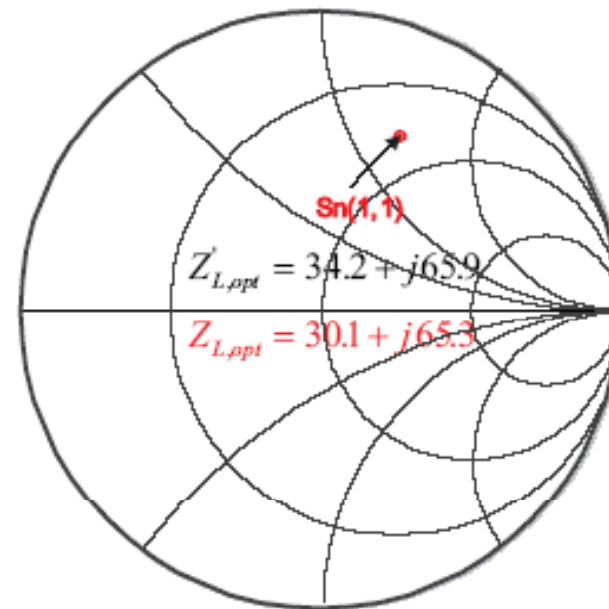
Matching Network Results (Output)

For the S-parameters ($Z_{L,opt}^*, 50$) at matched condition;

Similar to Input match; $S_{11} = 0$ and $|S_{21}| = 1$



Reference ($Z_{L,opt}^*, 50$)

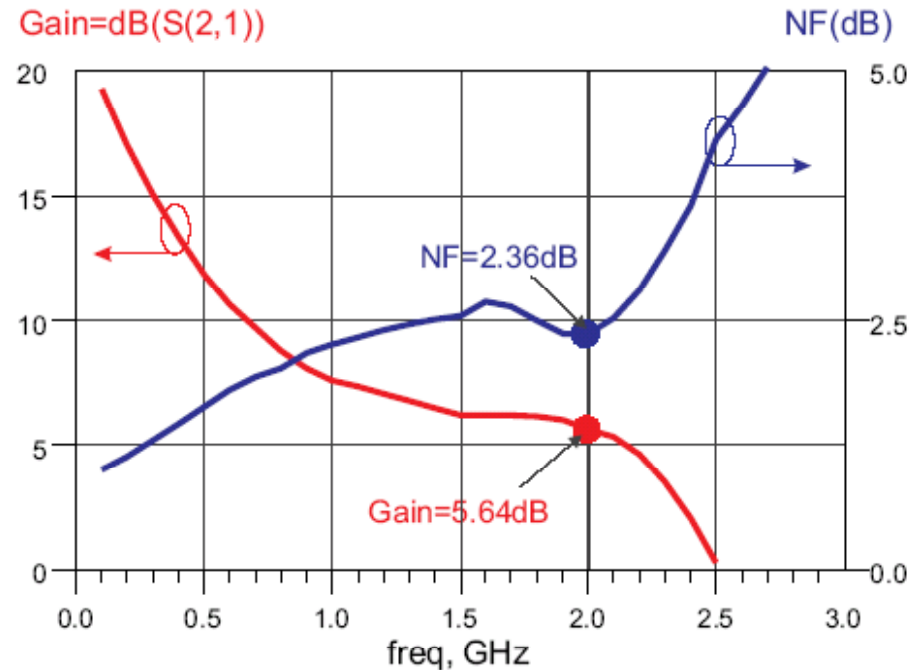


Reference (50,50)

Verification of Lumped Matching Networks

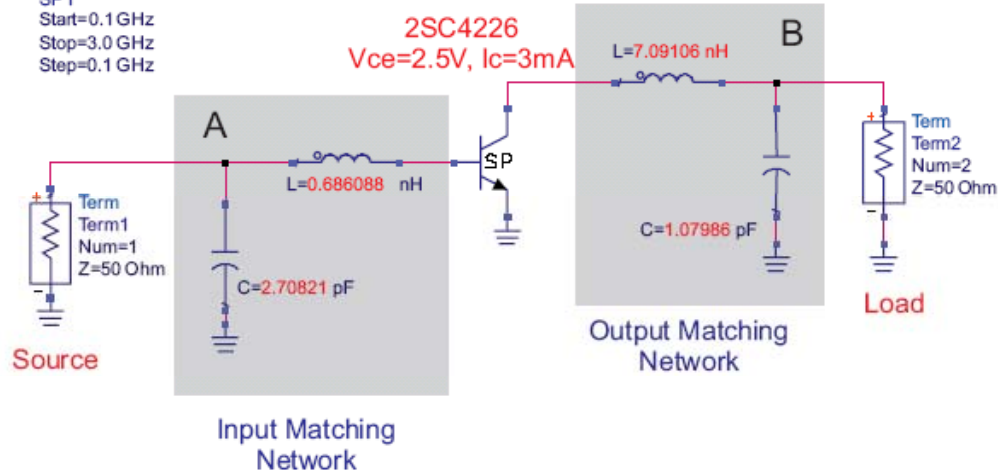
Gain @ 2GHz = 5.6dB

NF@2GHz = 2.36dB



S-PARAMETERS

S_Param
SP1
Start=0.1 GHz
Stop=3.0 GHz
Step=0.1 GHz



DC Bias Circuit for BJT

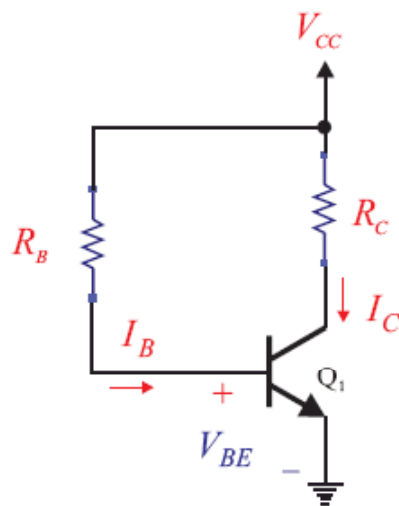
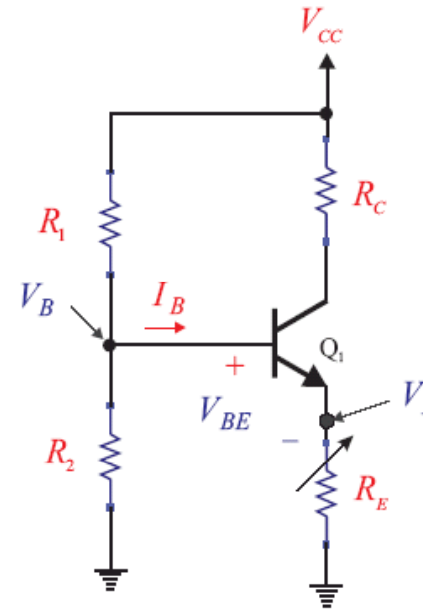
(1) Bias Circuit using Emitter Resistor

Neglecting I_B

$$V_B = \frac{R_2}{R_1 + R_2} V_{CC} \rightarrow$$

$$I_E = \frac{V_B - V_{BE}}{R_E} \cong I_C$$

- **Stable Bias,**
- **but Imperfect Emiiter GND**



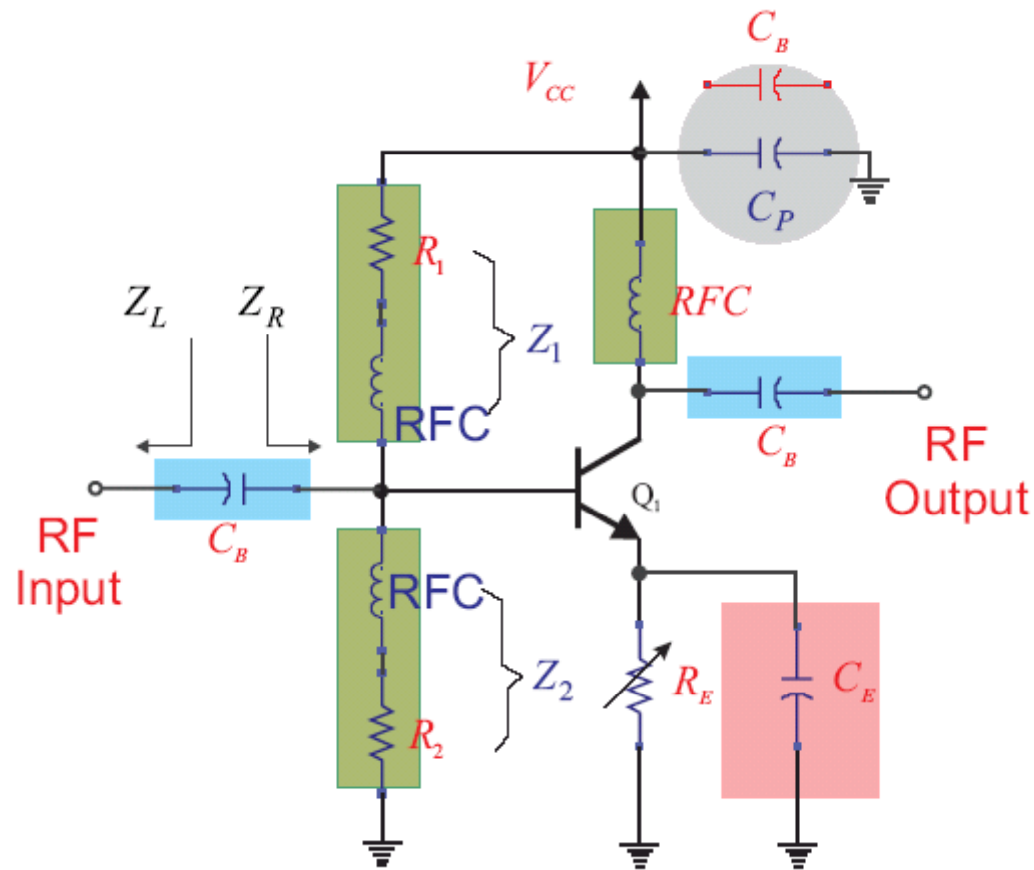
(2) Grounded Emitter Bias Circuit

$$I_B = \frac{V_{CC} - V_{BE}}{R_B} \rightarrow$$

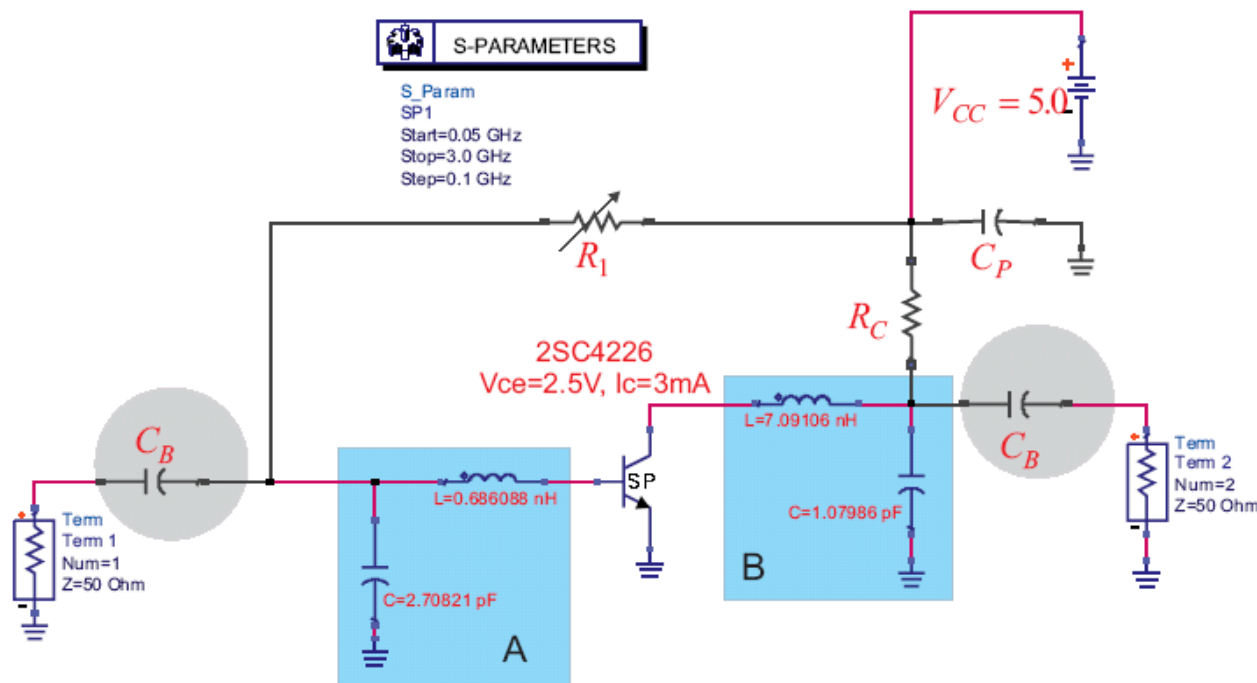
$$I_C = \beta I_B$$

- **Unstable Bias** $\leftarrow \beta$
- **Perfect Emiiter GND**

Decoupling and DC Blocking



Amplifier Circuit with DC Bias



Capacitors

$$C_B = 10 \text{ pF}$$

($\approx 1\Omega$ @ 2GHz)

$$C_p = 100 \text{ pF}$$

Resistors

$$R_1 = \frac{V_{CC} - V_{BE}}{I_B} = 143K$$

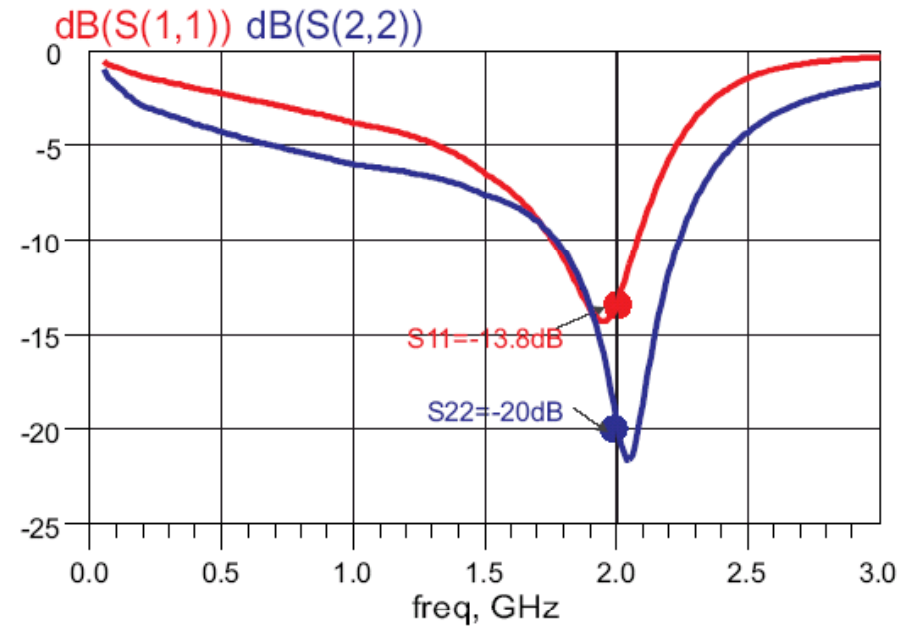
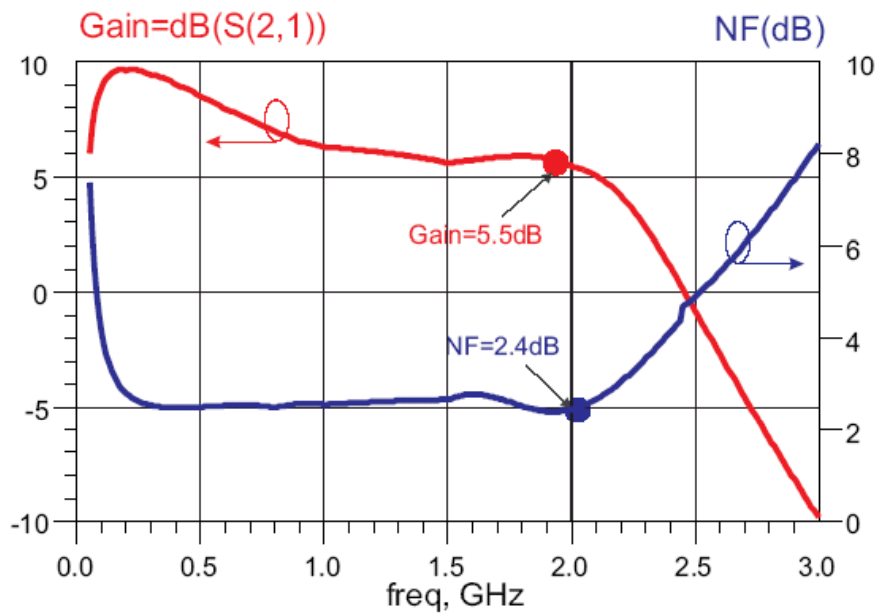
$$V_{CE} = 2.5$$

$$R_C = \frac{V_{CC} - V_{CE}}{I_C} = 833\Omega$$

$$(R_C \gg 50 \parallel 50 = 25)$$

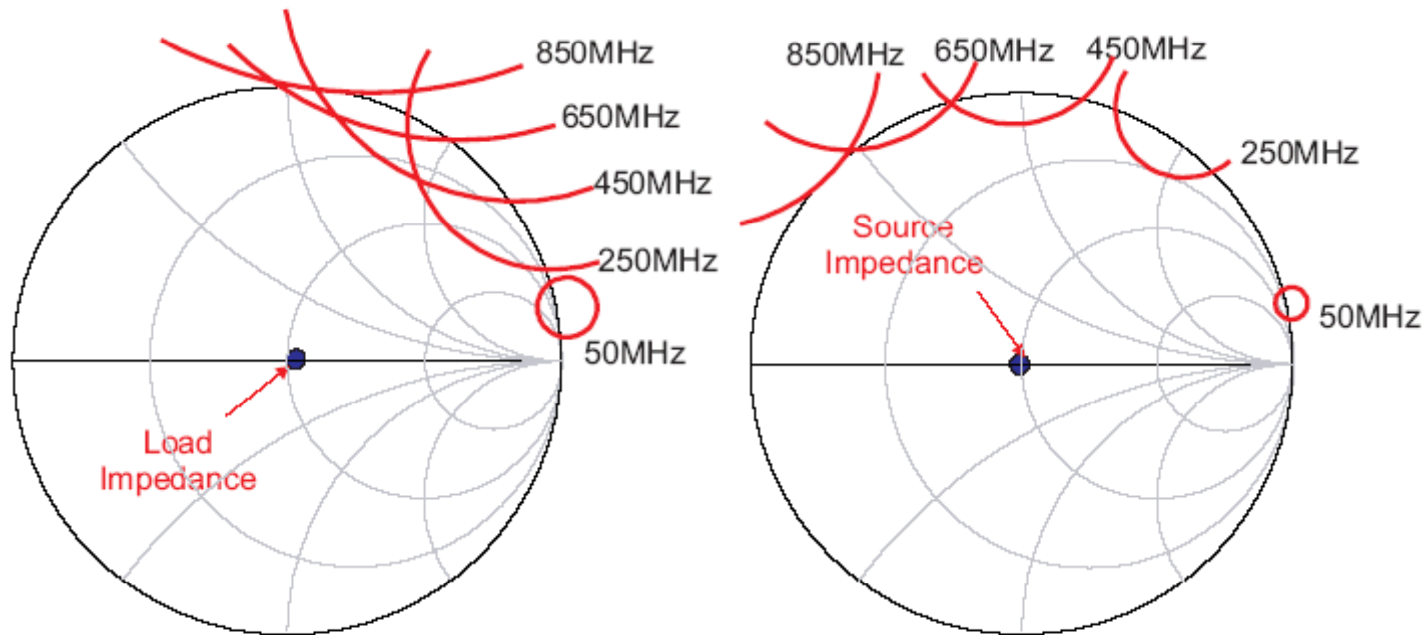
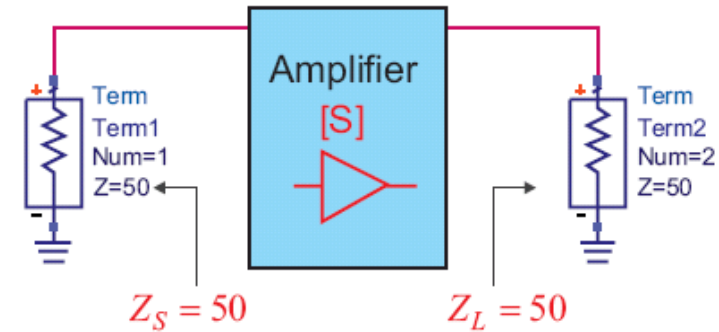
Performance of Designed Amplifier

- Gain=5.5dB>5dB, NF=2.4dB<3dB @2GHz
- Input RL=13.8dB, Output RL= 20dB



Source & load Stability in Low Frequency

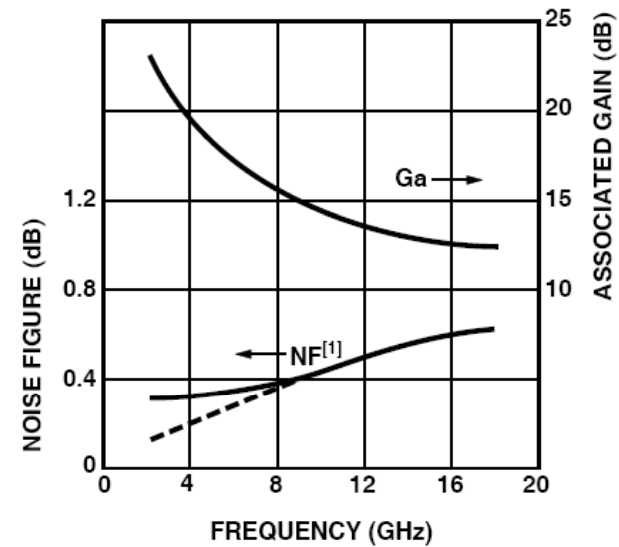
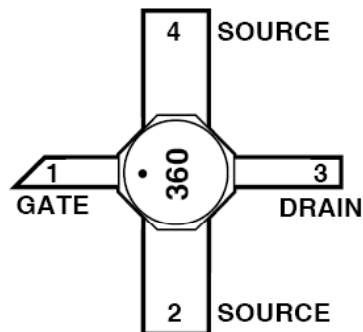
- Stable @ low frequency



Distributed LNA Example

Transistor (ATF36077)

NO.	Spec Item	Value
1	Frequency Range	9.7~10.3 GHz
2	DC Supply Voltage	3.3 V
3	DC Current Consumption	< 30 mA
4	Gain	> 10 dB
5	Noise Figure	< 1.5 dB



Substrate

- UL2000
 - Permittivity;2.5
 - Thickness;0.25mm(10 mil)
 - Conductor Thickness; 17um
 - Loss Tangent;0.0022

[1] Rogers corporation, ULTRALAM2000 HF laminate, Available at <http://www.rogerscorporation.com>

S-parameter input check

S-PARAMETERS

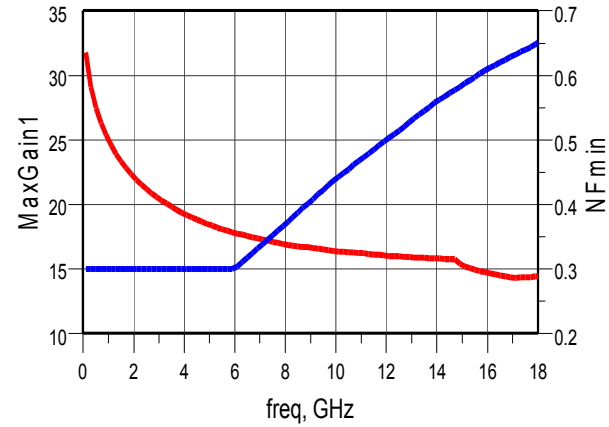
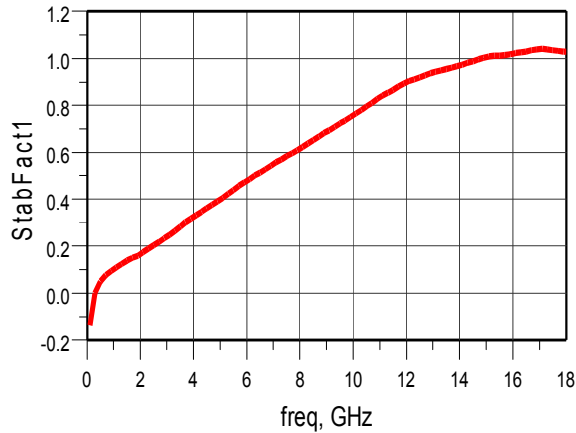
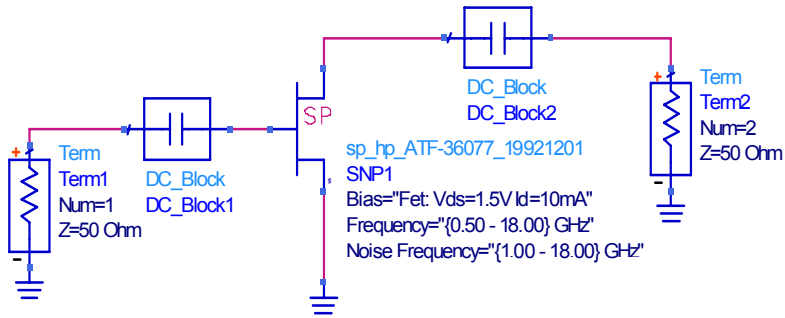
S_Param
 SP1
 Start=0.1 GHz
 Stop=18 GHz
 Step=0.2 GHz



StabFact
 StabFact1
 StabFact1=stab_fact(S)



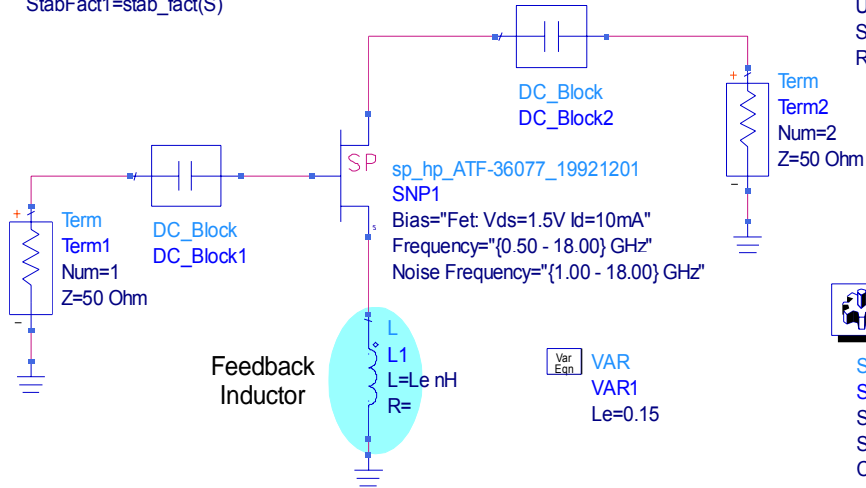
MaxGain
 MaxGain1
 MaxGain1=max_gain(S)



Stabilizing

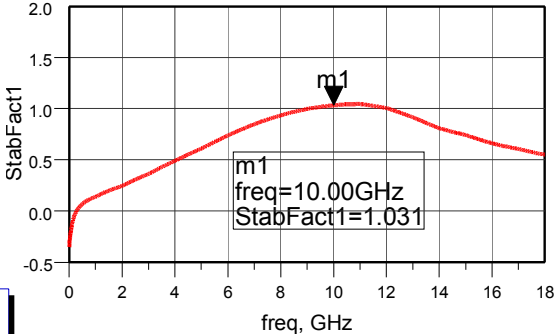


StabFact
StabFact1
StabFact1=stab_fact(S)



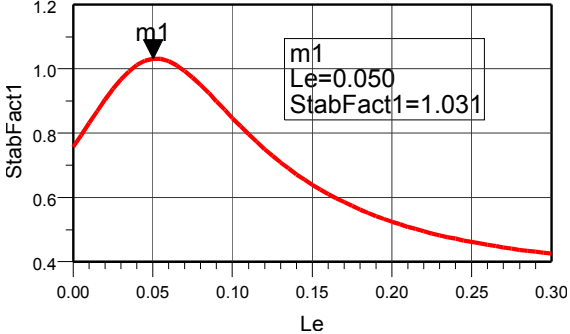
SWEEP PLAN

SweepPlan
SwpPlan1
Start=0 Stop=0.3 Step=0.005 Lin=
UseSweepPlan=
SweepPlan=
Reverse=no

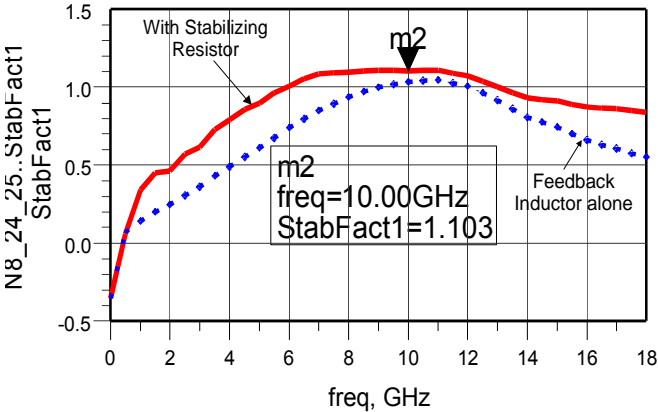
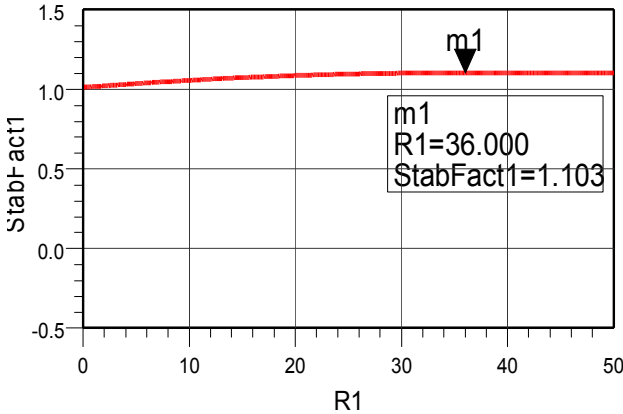
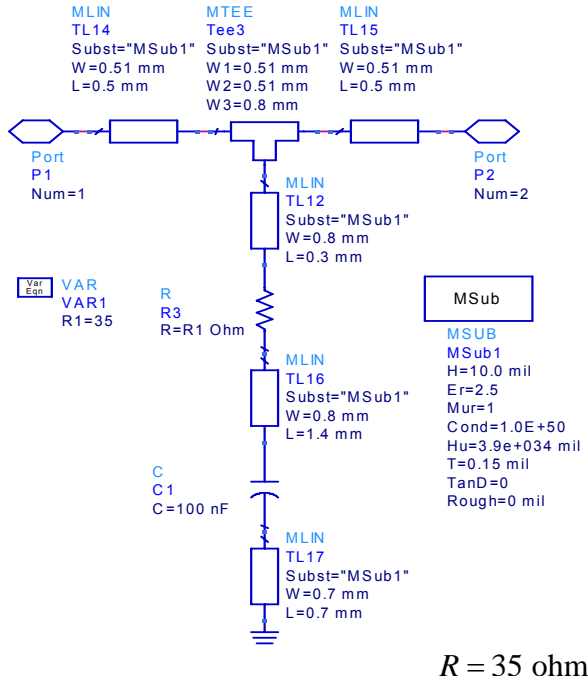


S-PARAMETERS

S_Param
SP1
SweepVar="Le"
SweepPlan="SwpPlan1"
CalcNoise=yes
Freq=10 GHz



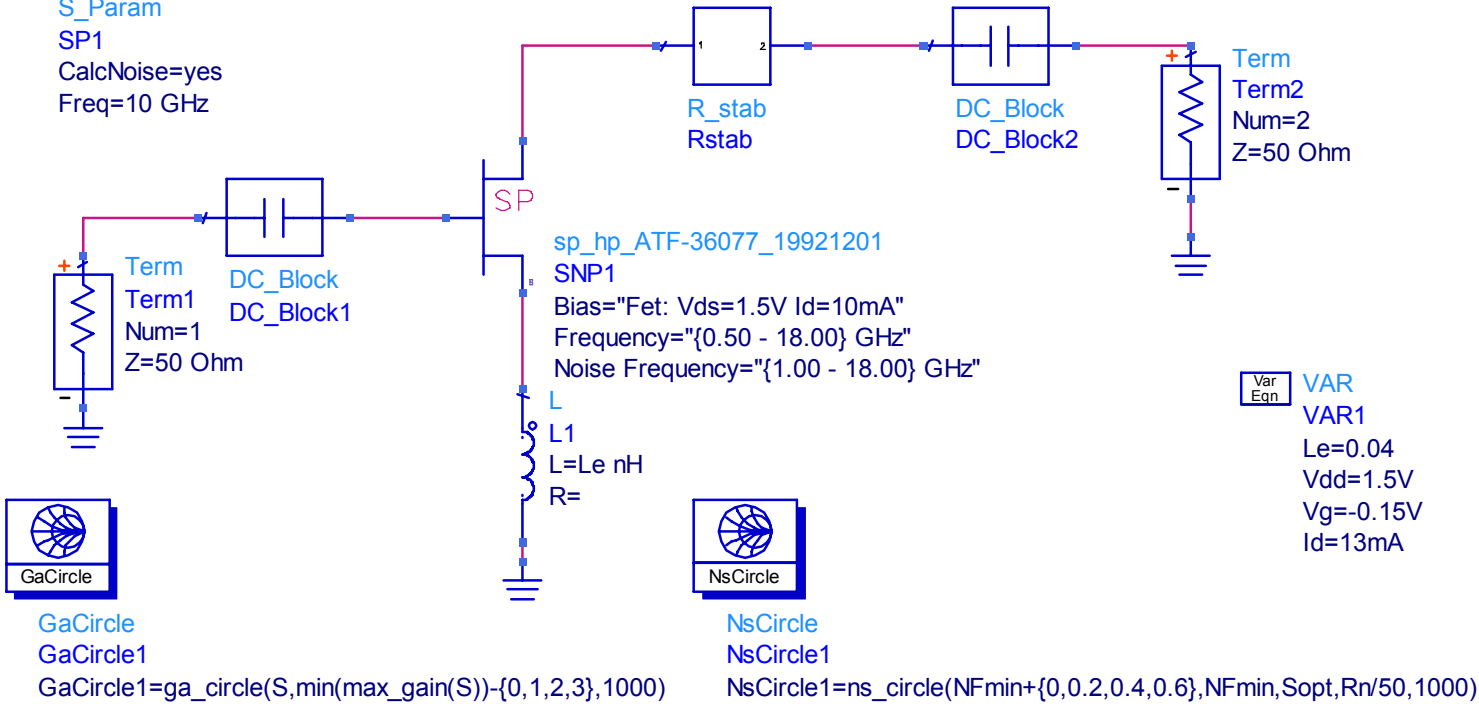
Further stabilization



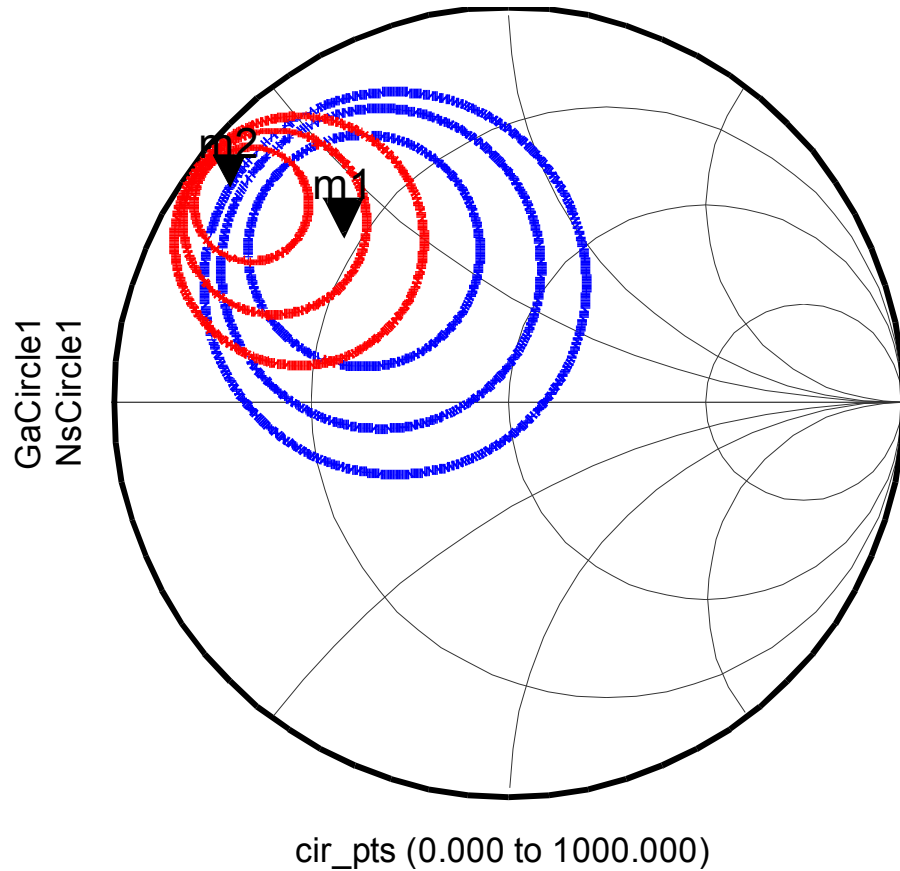
Input and Output Impedance Pick up

S-PARAMETERS

S_Param
 SP1
 CalcNoise=yes
 Freq=10 GHz



Circles



Choice m1

m1
 indep(m1)=1000
 NsCircle1=0.609 / 133.725
 ns figure=0.474788
 impedance = $Z_0 * (0.285 + j0.398)$

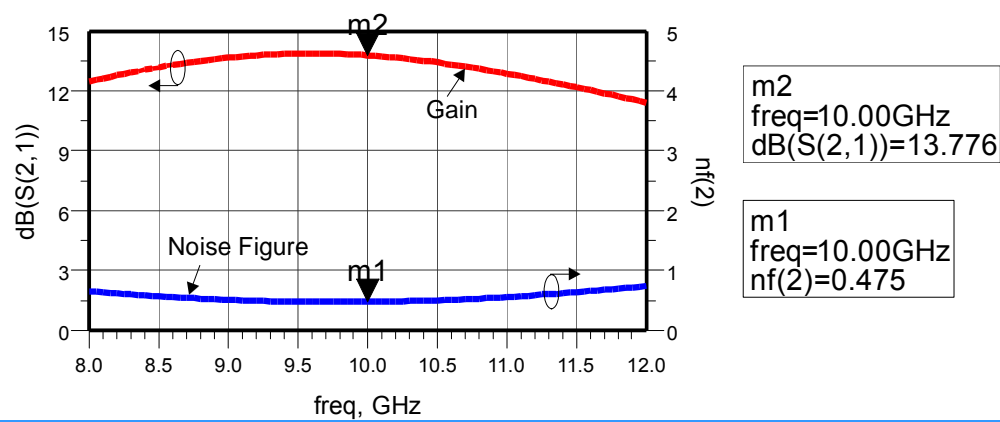
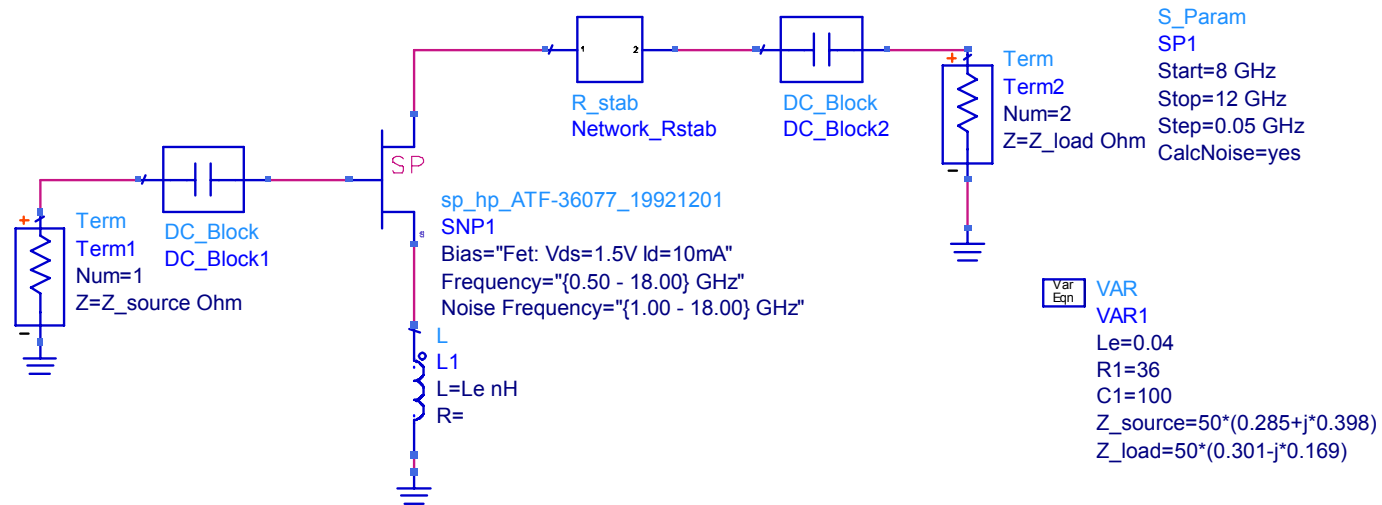
m2
 indep(m2)=1000
 GaCircle1=0.895 / 142.486
 gain=15.380759
 impedance = $Z_0 * (0.062 + j0.338)$

$$\Gamma_s = 0.609 \angle 133.725^\circ$$

$$\Gamma_L = \left(S_{22} + \frac{S_{12} S_{21} \Gamma_s}{1 - S_{11} \Gamma_s} \right)^* = 0.548 \angle -159.04^\circ$$

Verification

S-PARAMETERS



Matching Network Design

- Distributed Matching Network Design
 - Replacement
 - Verification and Comparison
- Bias Network Adding
 - DC Block
 - RF choke
 - Unstability at Low Frequency

Matching Network Design(Input)

OPTIM

Optim
 Optim1
 SaveOptimVars=no
 UpdateDataset=yes
 UseAllOptVars=yes
 UseAllGoals=yes

GOAL

Goal
 OptimGoal1
 Expr="mag(S21)"
 SimInstanceName="SP1"
 Min=0.9999
 Max=1.0001
 Weight=
 RangeVar[1]=
 RangeMin[1]=
 RangeMax[1]=

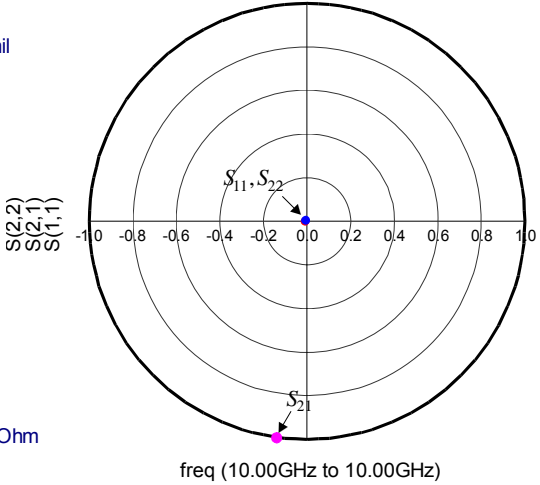
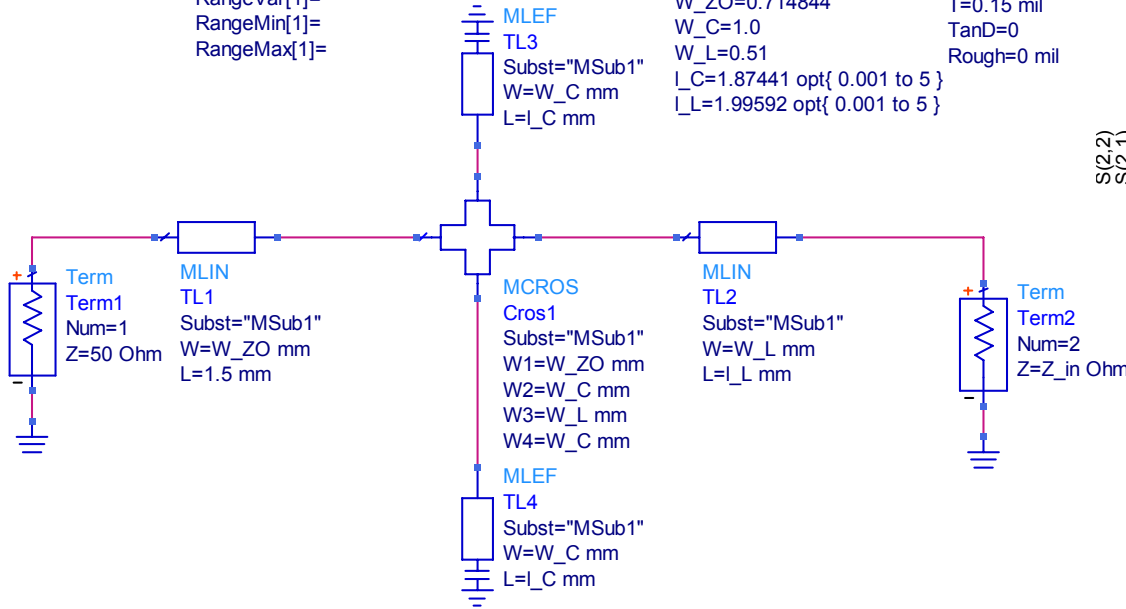
S-PARAMETERS

S_Param
 SP1
 Freq=10 GHz

Var Eqn
 VAR
 VAR1
 $Z_in=50*(0.285-j*0.398)$
 $W_ZO=0.714844$
 $W_C=1.0$
 $W_L=0.51$
 $I_C=1.87441 \text{ opt}\{0.001 \text{ to } 5\}$
 $I_L=1.99592 \text{ opt}\{0.001 \text{ to } 5\}$

MSub

MSUB
 MSub1
 H=0.25 mm
 Er=2.5
 Mur=1
 Cond=1.0E+50
 Hu=3.9e+034 mil
 T=0.15 mil
 TanD=0
 Rough=0 mil



Matching Network Design(Output)

OPTIM

Optim1
 SaveOptimVars=no
 UpdateDataset=yes
 UseAllOptVars=yes
 UseAllGoals=yes

GOAL

Goal
 OptimGoal1
 Expr="mag(S21)"
 SimInstanceName="SP1"
 Min=0.9999
 Max=1.0001
 Weight=
 RangeVar[1]=
 RangeMin[1]=
 RangeMax[1]=

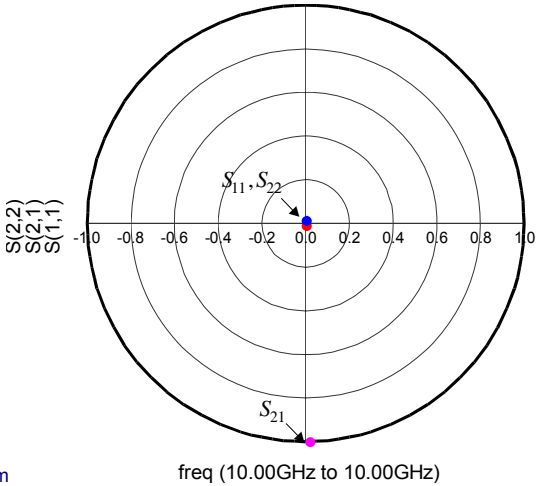
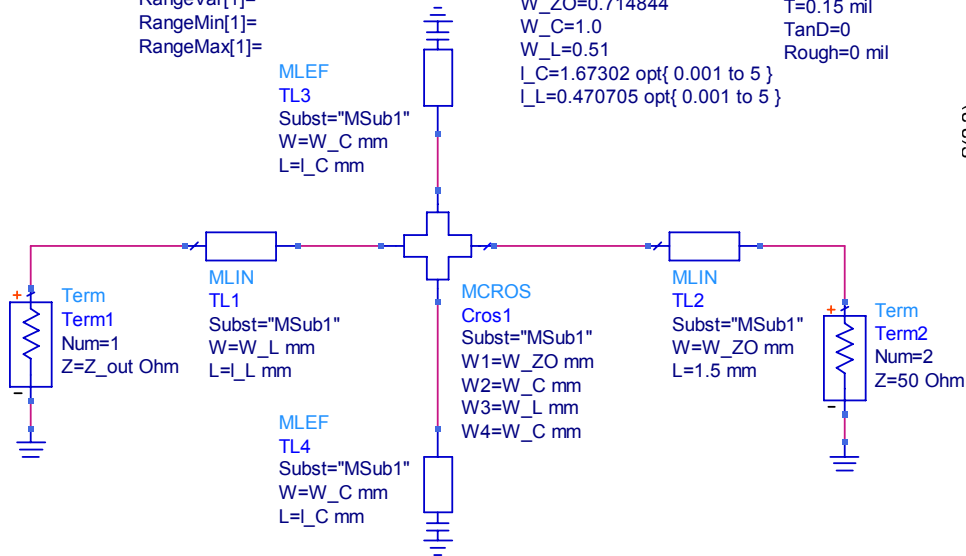
S-PARAMETERS

S_Param
 SP1
 Freq=10 GHz

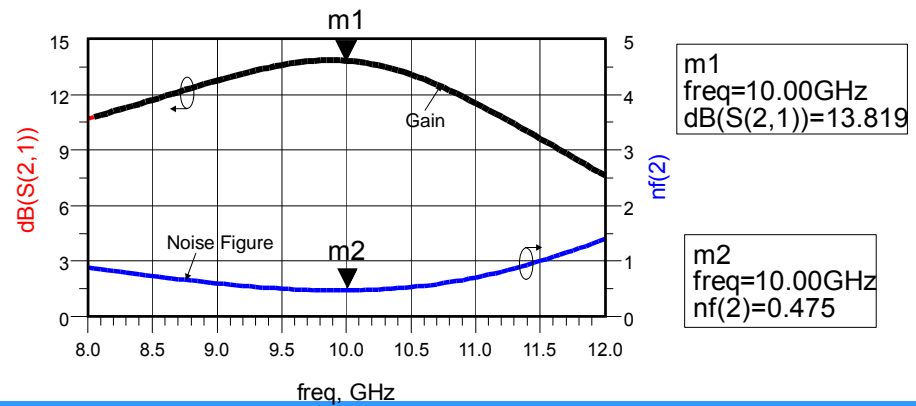
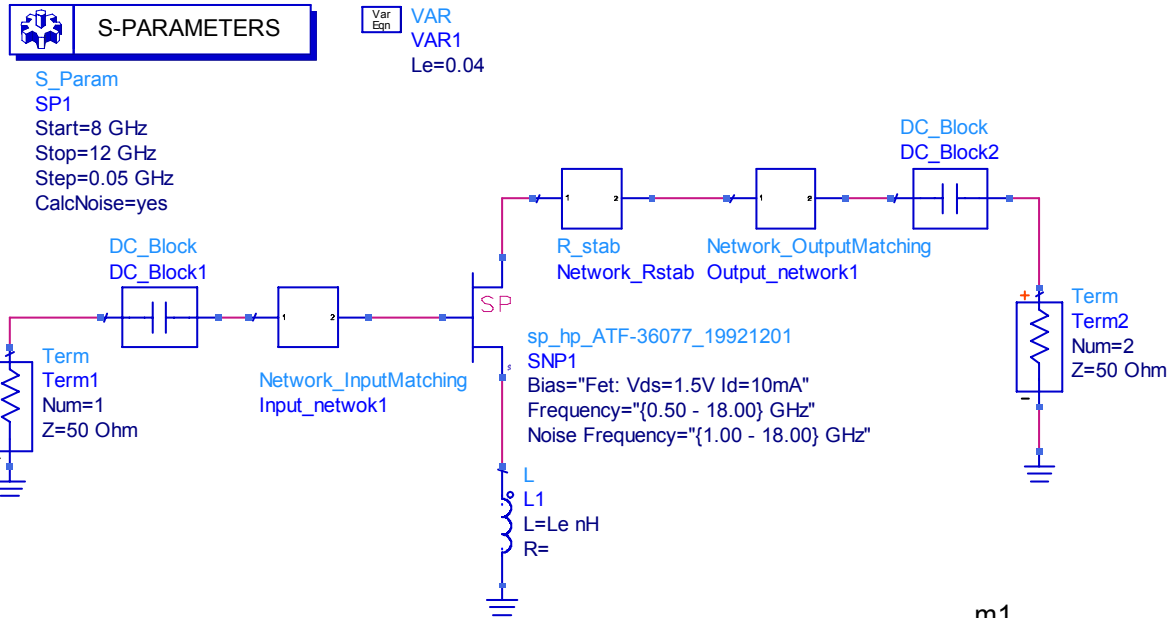
VAR
 VAR1
 $Z_{out}=50*(0.300+j*0.167)$
 $W_{ZO}=0.714844$
 $W_C=1.0$
 $W_L=0.51$
 $I_C=1.67302 \text{ opt}\{0.001 \text{ to } 5\}$
 $I_L=0.470705 \text{ opt}\{0.001 \text{ to } 5\}$

MSub

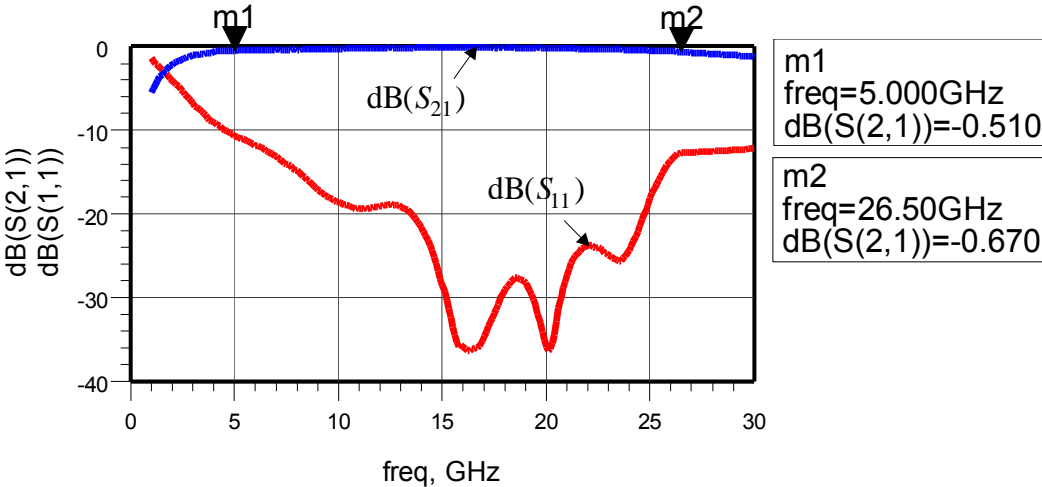
MSUB
 MSub1
 H=0.25 mm
 Er=2.5
 Mur=1
 Cond=1.0E+50
 Hu=3.9e+034 mil
 T=0.15 mil
 TanD=0
 Rough=0 mil



Verification of Lumped Matching Networks

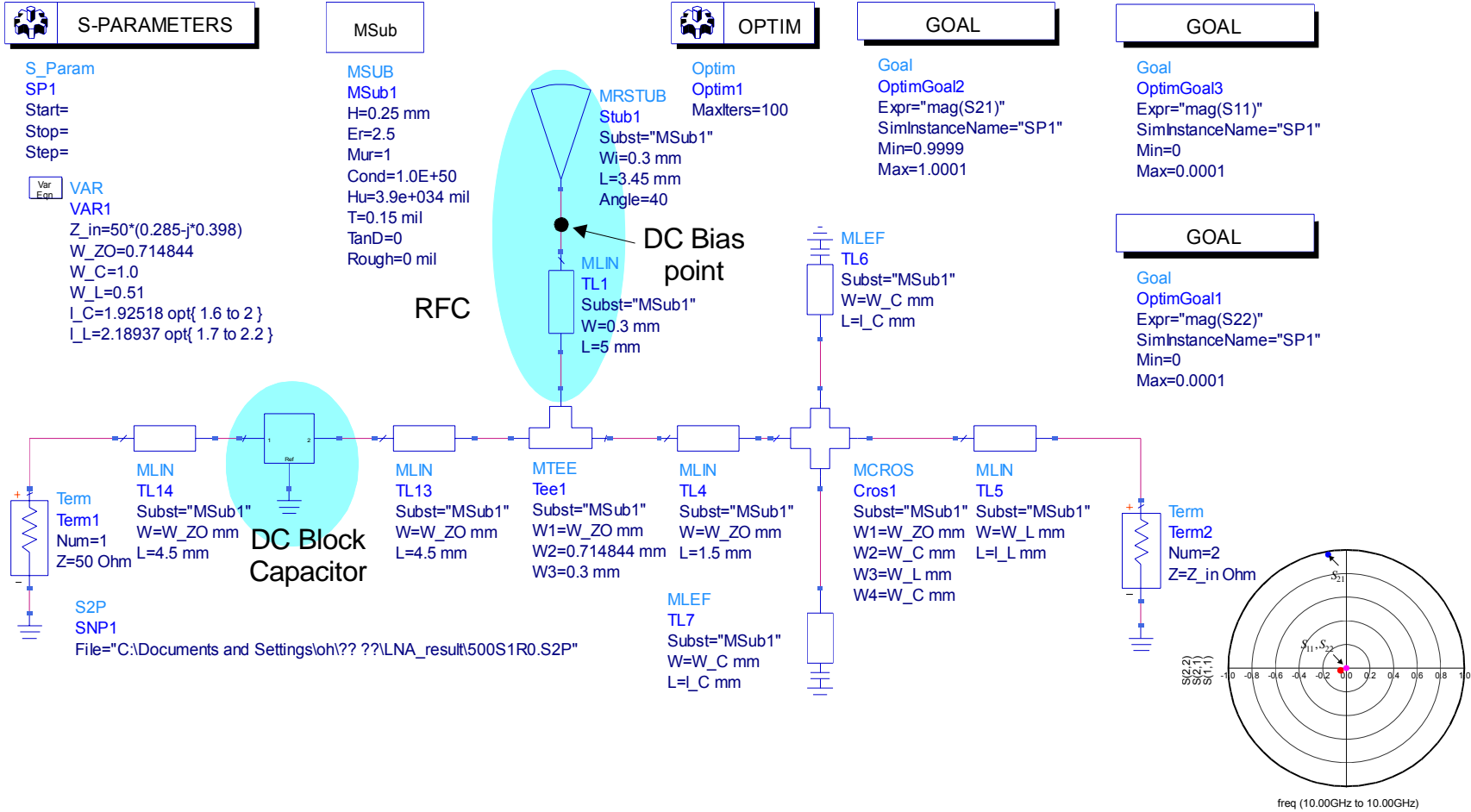


DC Block Capacitor



[2] American Technical Ceramics, ATC 545L Series UBC Ultra-Broadband Capacitor, Available at <http://www.atceramics.com>

Bias Network Adding(Input)



Bias Network Adding(Output)

OPTIM

Optim
Optim1
SaveOptimVars=no
UpdateDataset=yes
UseAllOptVars=yes
UseAllGoals=yes

GOAL

Goal
OptimGoal3
Expr="mag(S22)"
SimInstanceName="SP1"
Min=0
Max=0.0001

GOAL

Goal
OptimGoal2
Expr="mag(S11)"
SimInstanceName="SP1"
Min=0
Max=0.0001

GOAL

Goal
OptimGoal1
Expr="mag(S21)"
SimInstanceName="SP1"
Min=0.9999
Max=1.0001

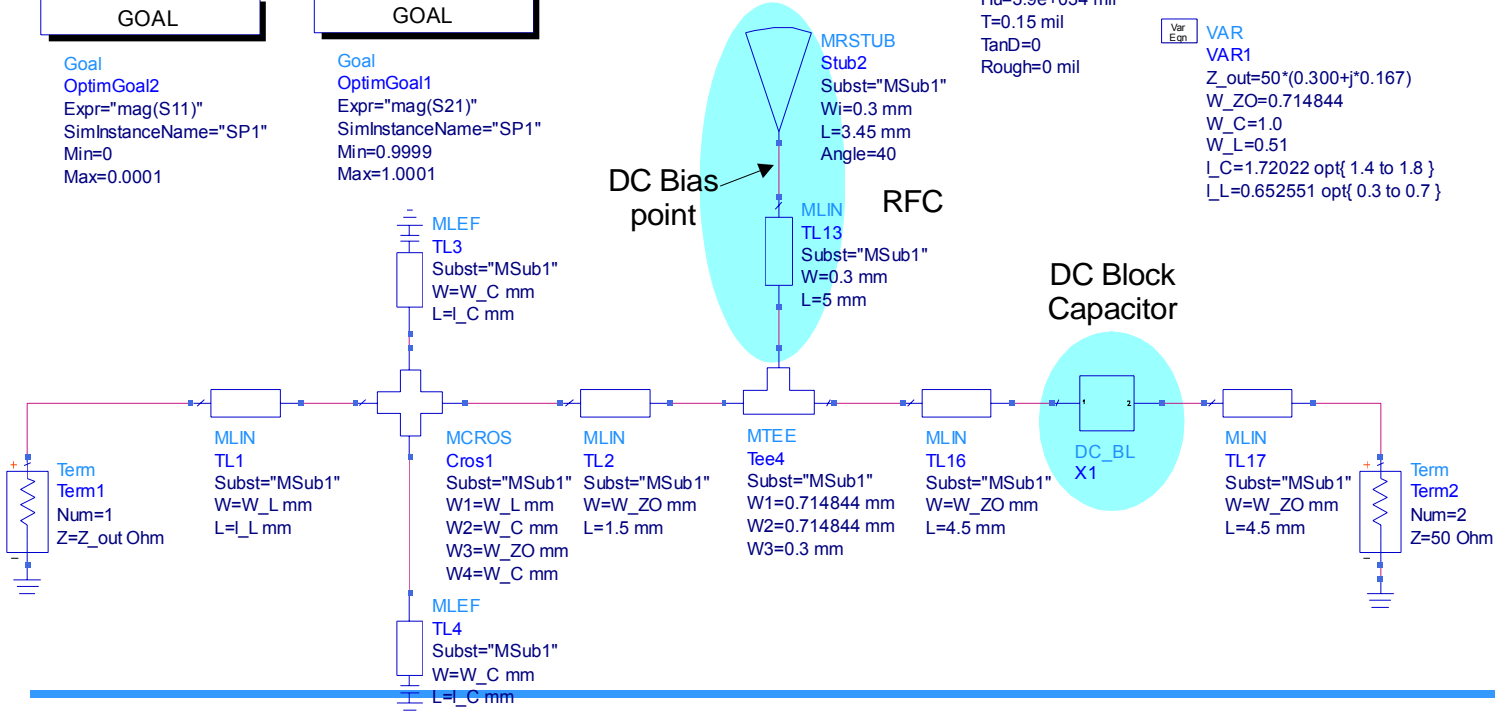
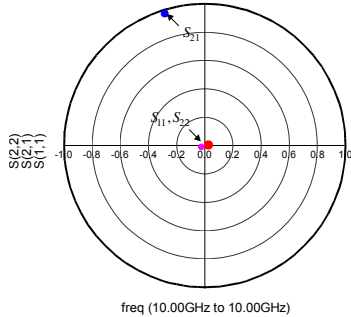
MSub

MSub
MSub
H=0.25 mm
Er=2.5
Mur=1
Cond=1.0E+50
Hu=3.9e+034 mil
T=0.15 mil
TanD=0
Rough=0 mil

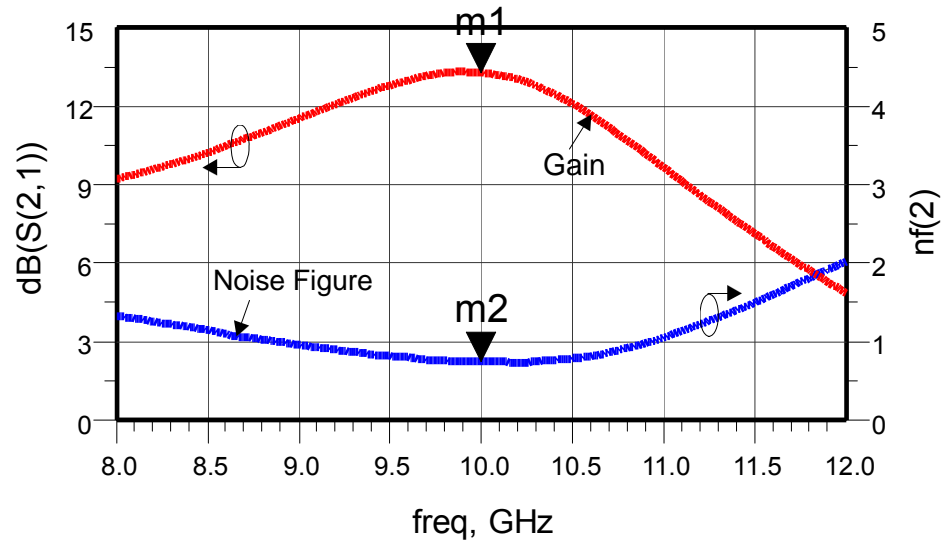
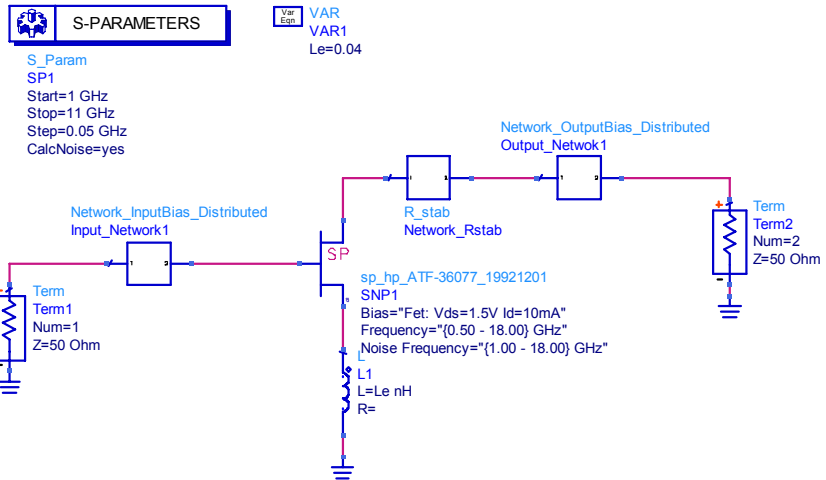
S-PARAMETERS

S_Param
SP1
Freq=10 GHz

Var
Eqn
VAR
VAR1
Z_out=50*(0.300+j*0.167)
W_ZO=0.714844
W_C=1.0
W_L=0.51
I_C=1.72022 opt{ 1.4 to 1.8 }
I_L=0.652551 opt{ 0.3 to 0.7 }



Results

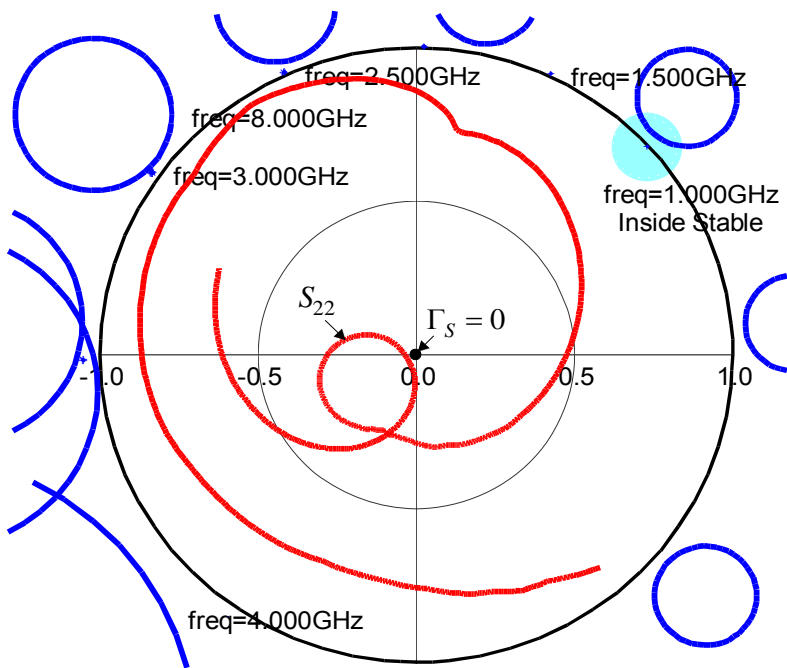


m1
 freq=10.00GHz
 dB(S(2,1))=13.269

m2
 freq=10.00GHz
 nf(2)=0.745

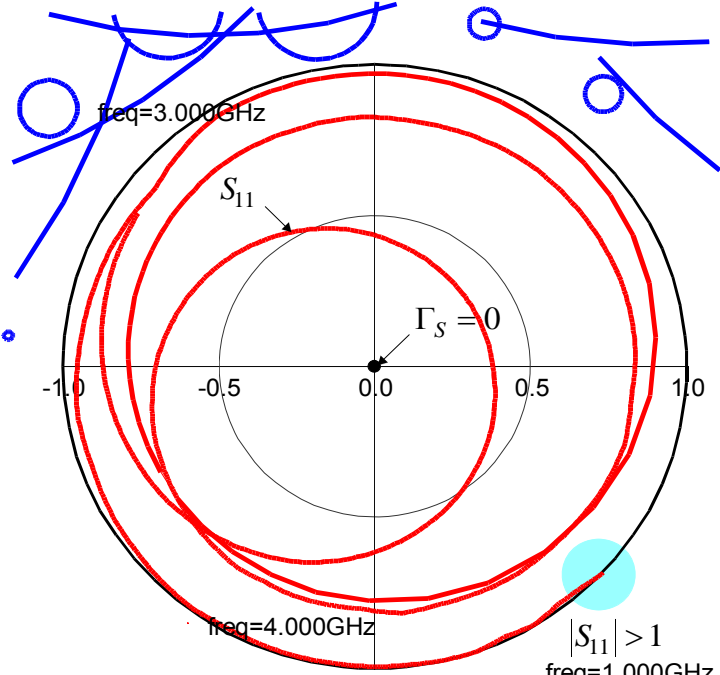
Source and Load Stabilities in Low Frequency

Eqn Source_Stability=s_stab_circle(S[0::50::800])



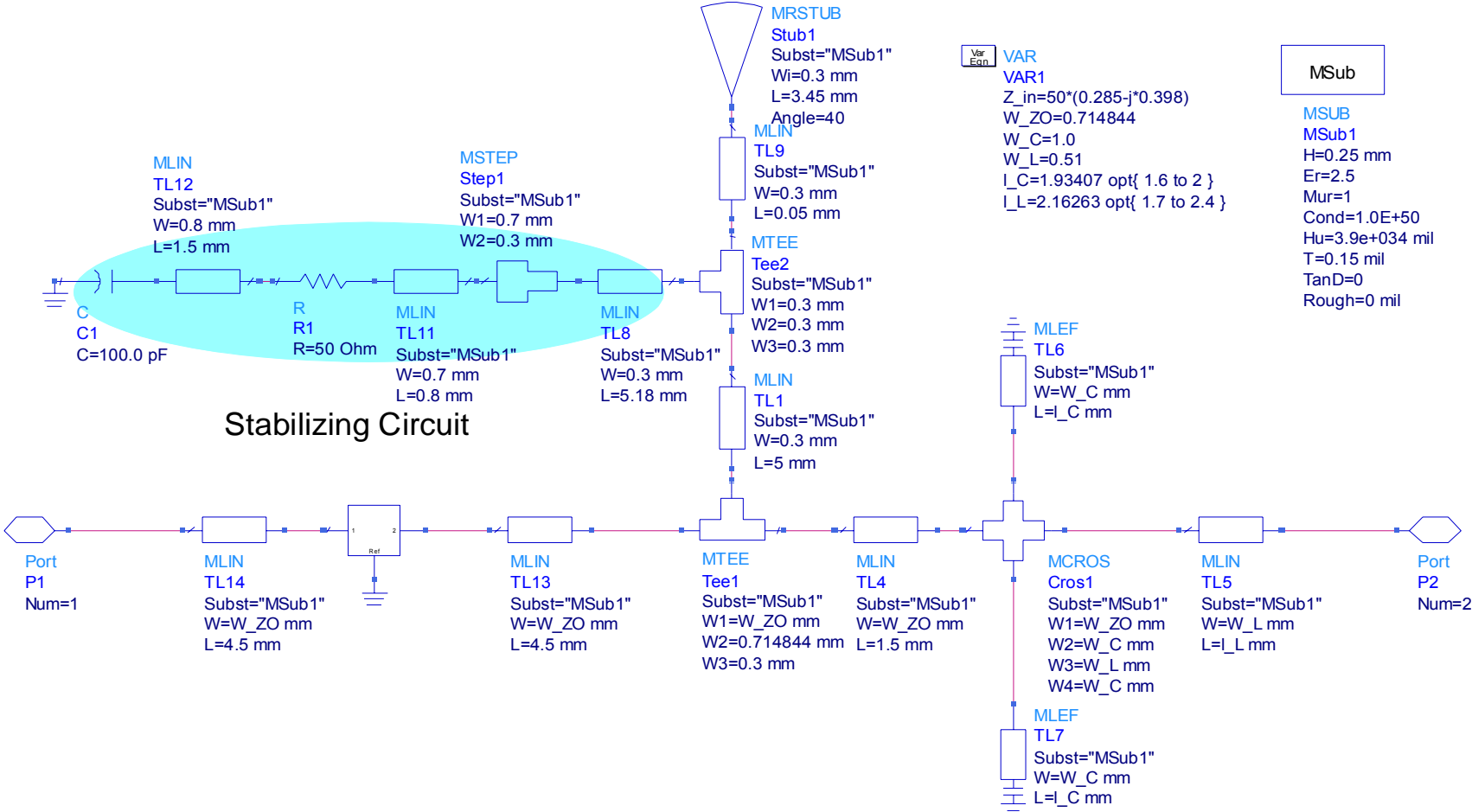
indep(Source_Stability) (0.000 to 51.000)
freq (1.000GHz to 12.00GHz)

Eqn Load_Stability=l_stab_circle(S[0::50::800])



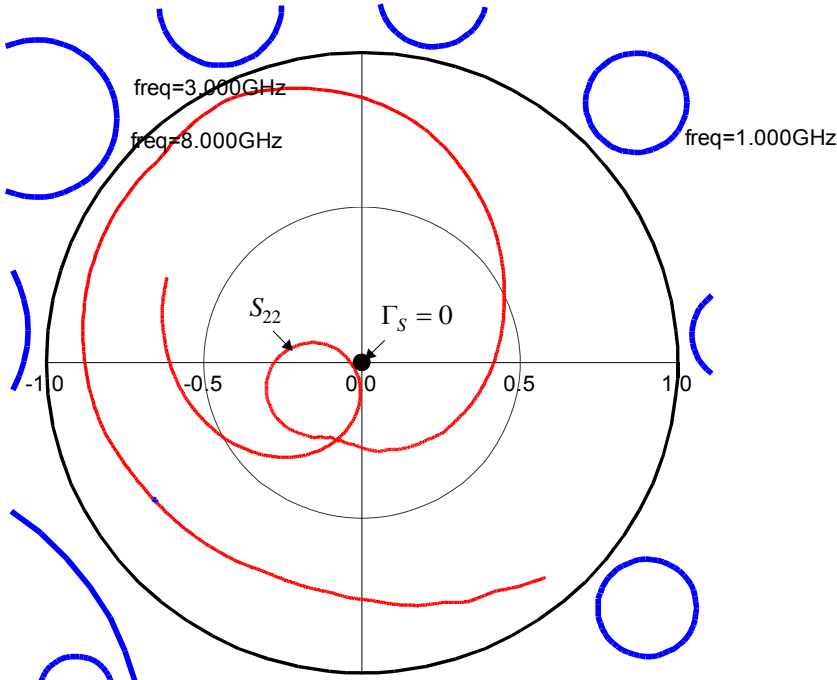
indep(Load_Stability) (0.000 to 51.000)
freq (1.000GHz to 12.00GHz)

Stability Improvement Circuit(Input)



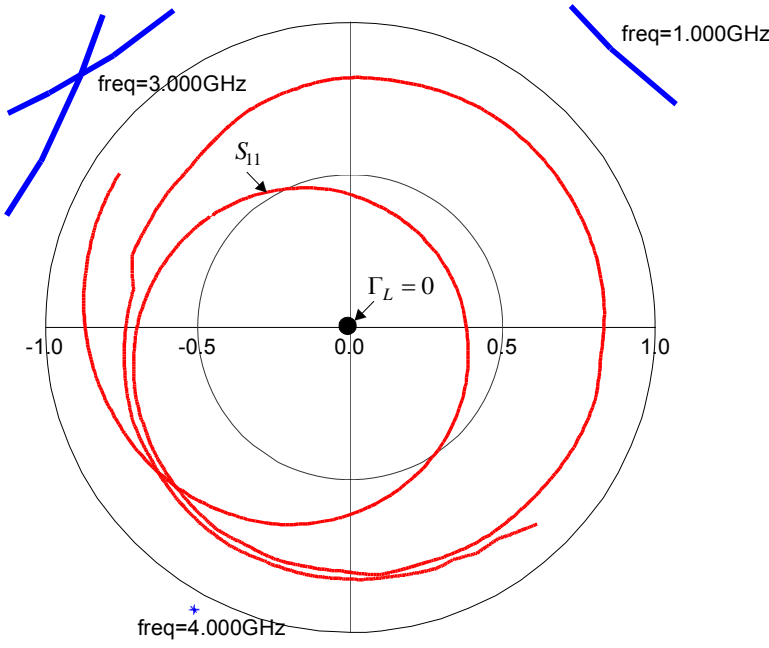
Results

Eqn Source_Stability=s_stab_circle(S[0::50::800])



indep(Source_Stability) (0.000 to 51.000)
freq (1.000GHz to 12.00GHz)

Eqn Load_Stability=l_stab_circle(S[0::50::800])

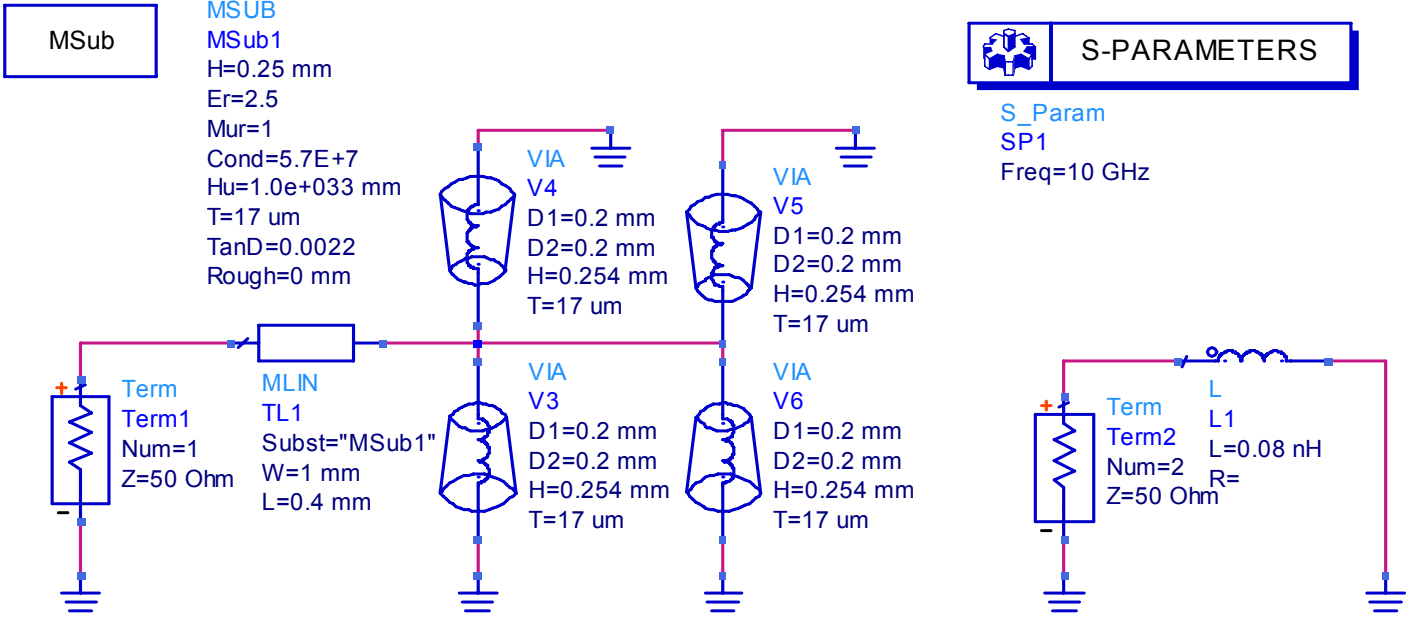


indep(Load_Stability) (0.000 to 51.000)
freq (1.000GHz to 12.00GHz)

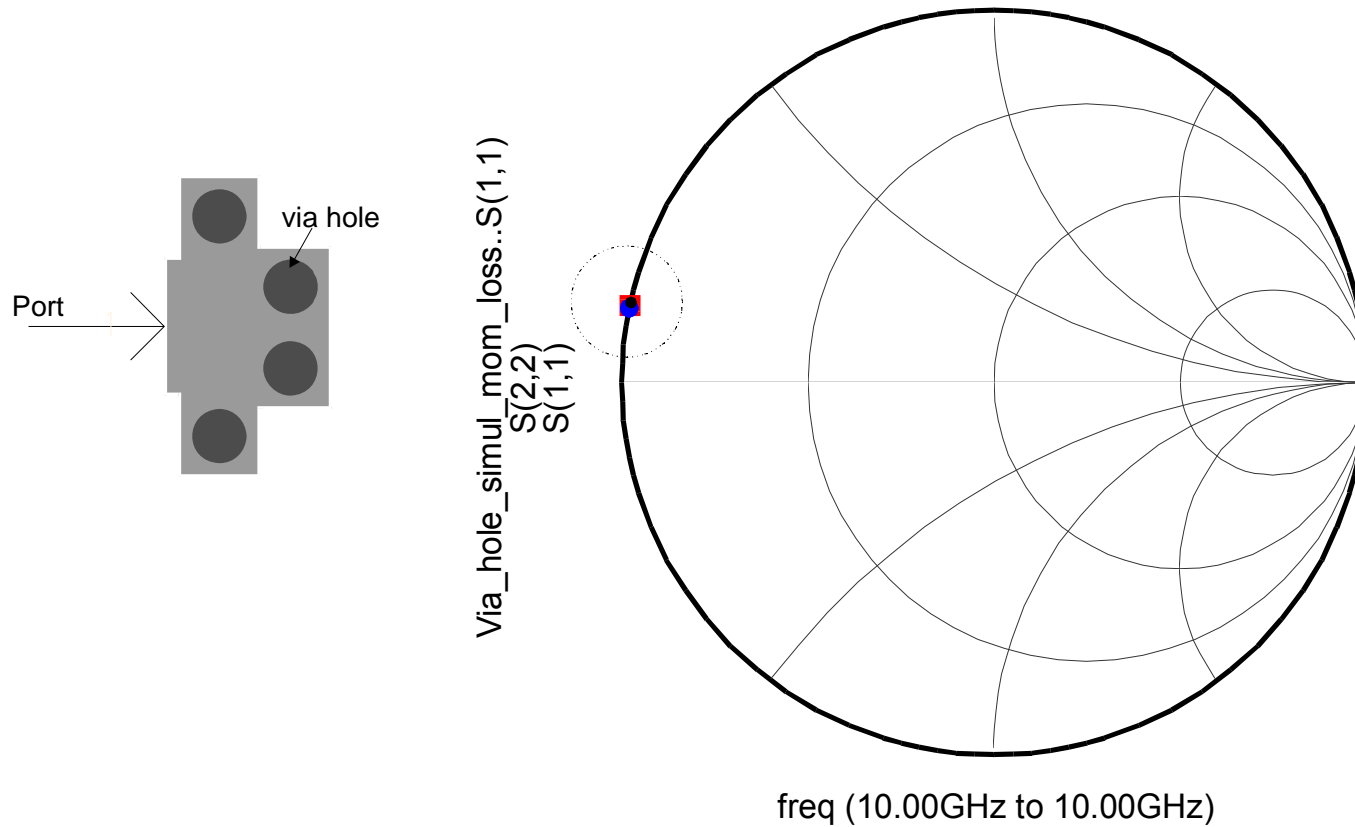
Layout

- AutoLayout
- Pattern Simulation
 - EM simulation and Trimming
- Manual Modification

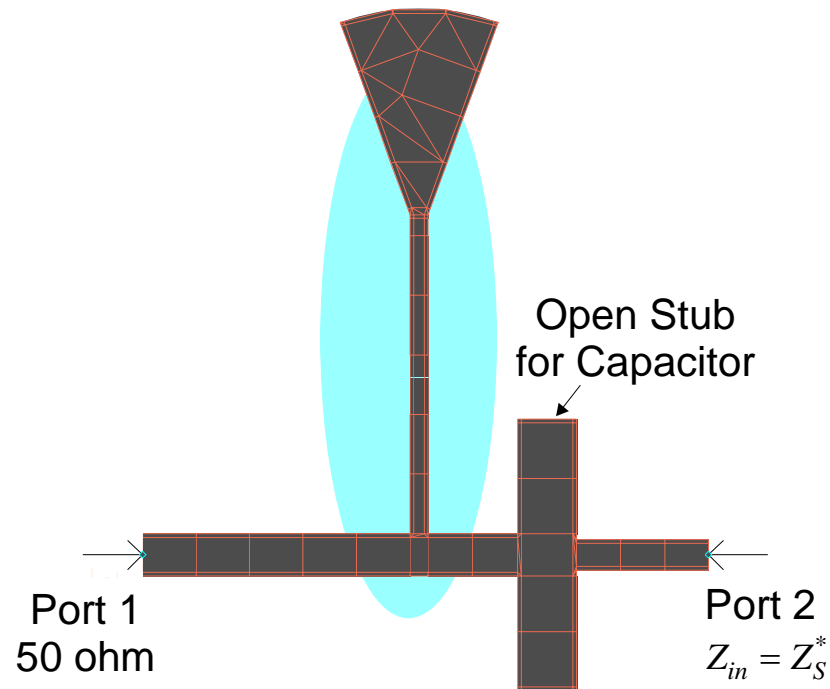
Source Inductance



EM Design of Source Inductance



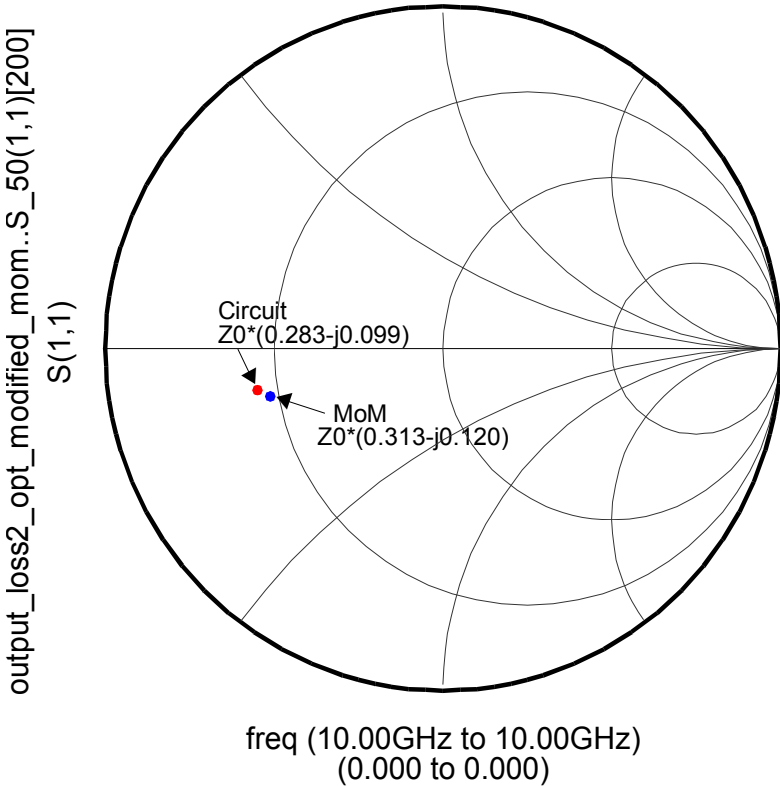
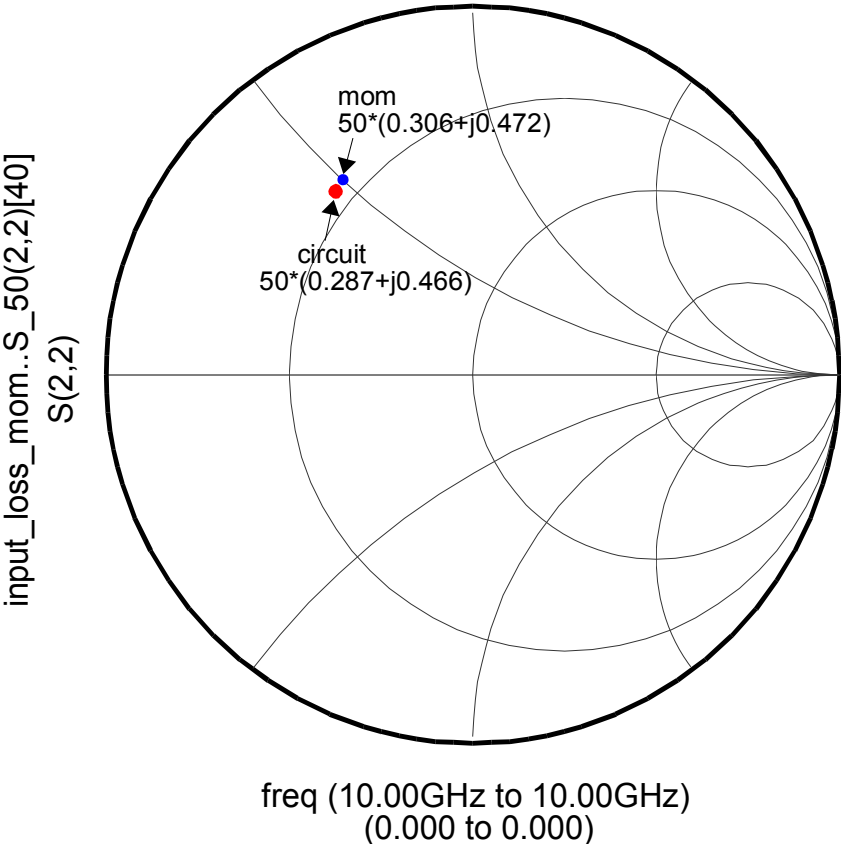
EM Tune of Matching Circuit



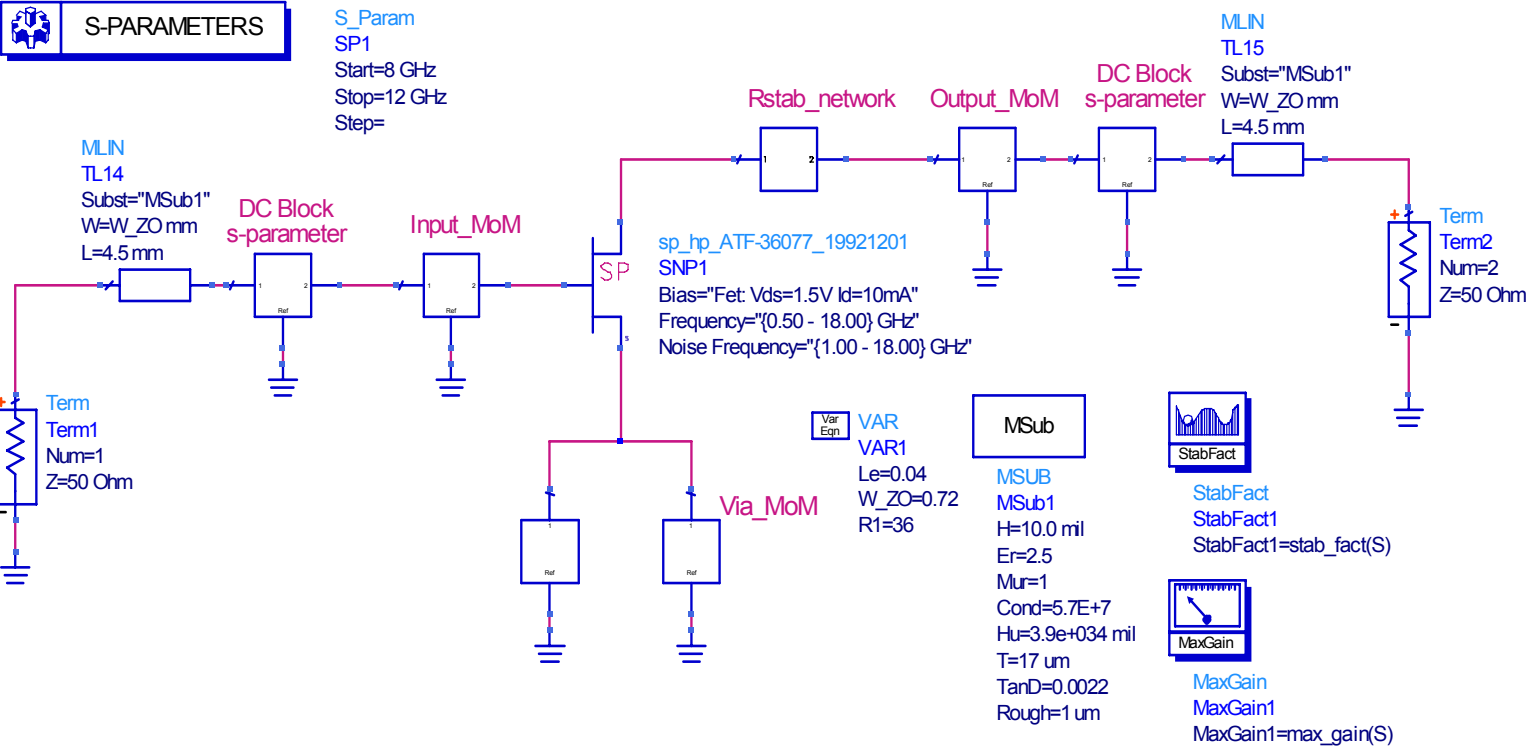
Stub length is tuned

DC Block capacitor excluded

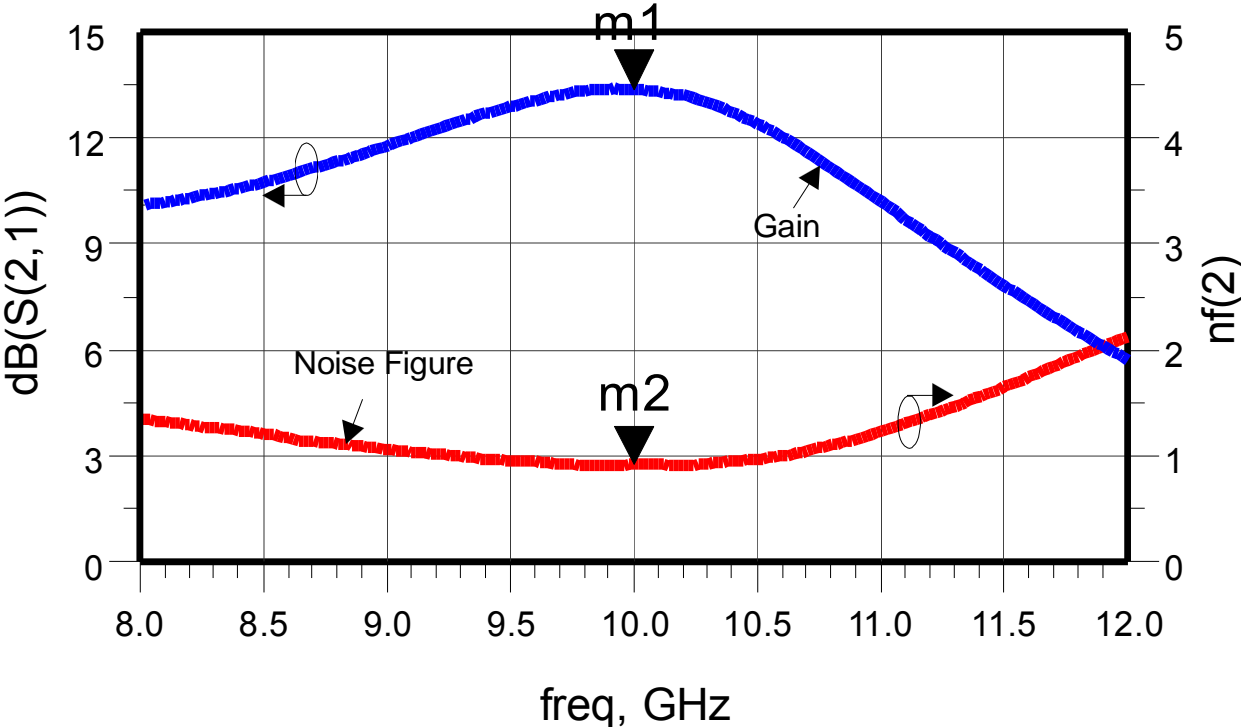
Results



EM Verification



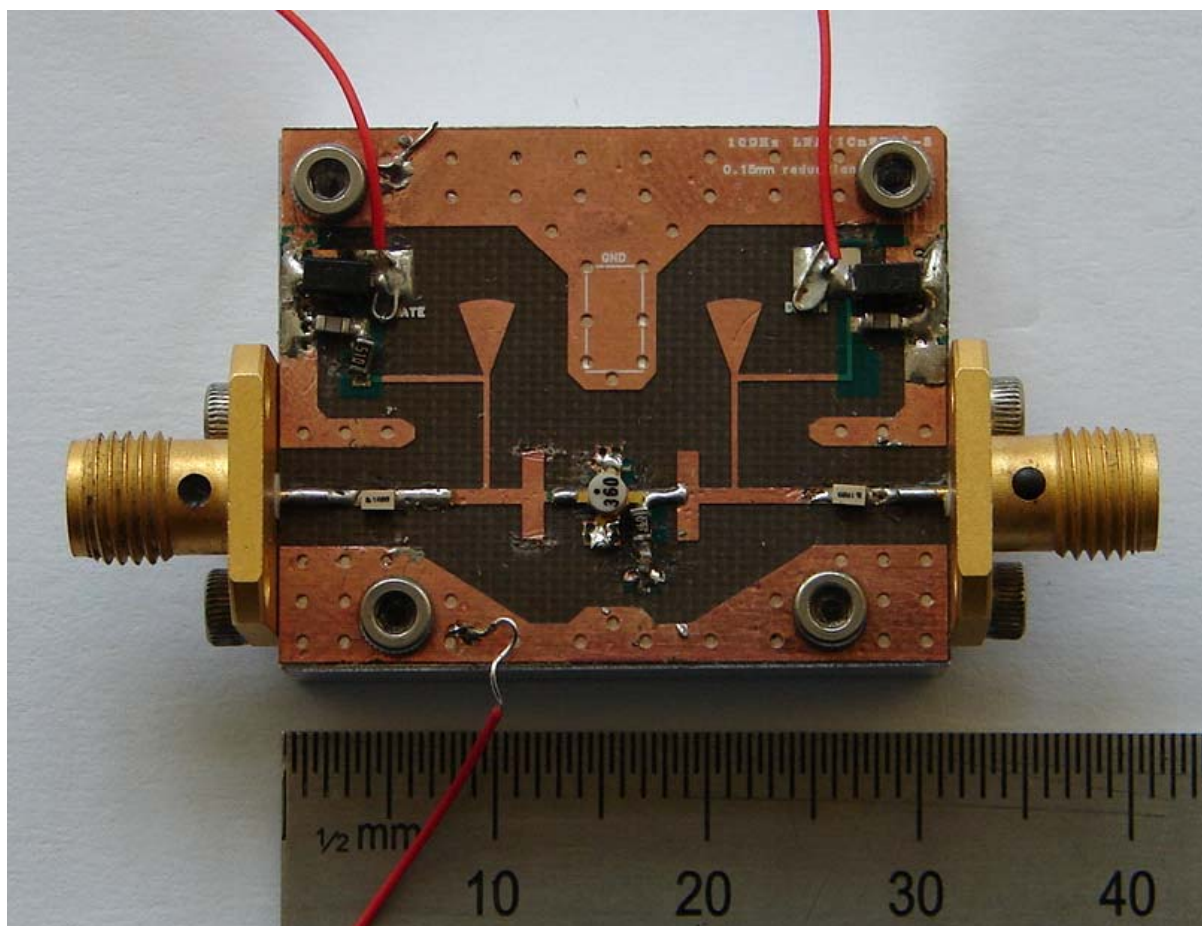
Results



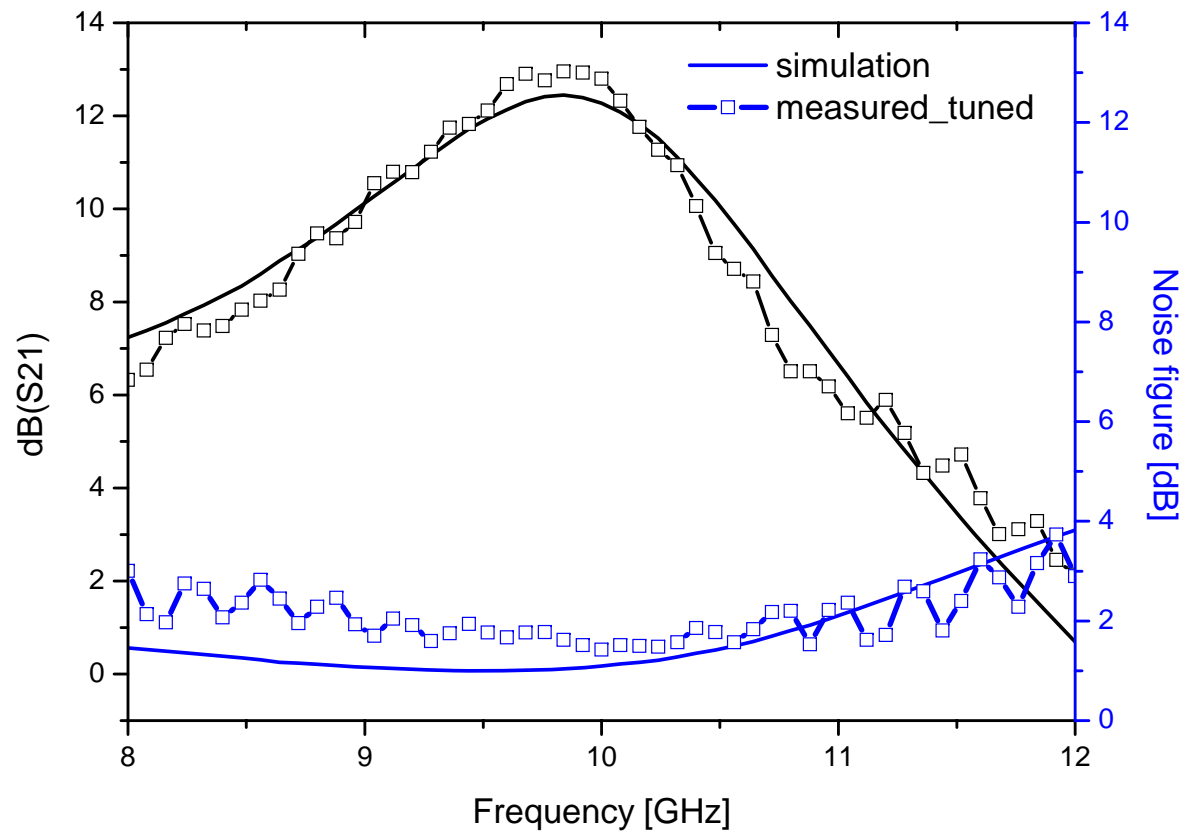
m1
freq= 10.00GHz
dB(S(2,1))=13.367

m2
freq= 10.00GHz
nf(2)=0.914

Fabrication



Measured Results



Reference

- Reference
 - [1]. Hewlett Packard, *HP85150B Microwave and RF design system*, 1995
 - [2]. G.D. Vendelin et al, *Microwave circuit design*, John and Wiley & Sons, 1990.
 - [3]. H. Fukui, *Low noise microwave transistors & amplifiers*, IEEE Press, 1981