TE 364

LECTURE 5

Resonant Circuits

2018.02.25

Abdul-Rahman Ahmed

Introduction

- Resonant circuits are used in many applications such as
  - Filters
  - Oscillators
  - Tuners
  - tuned amplifiers
  - and microwave communication networks.

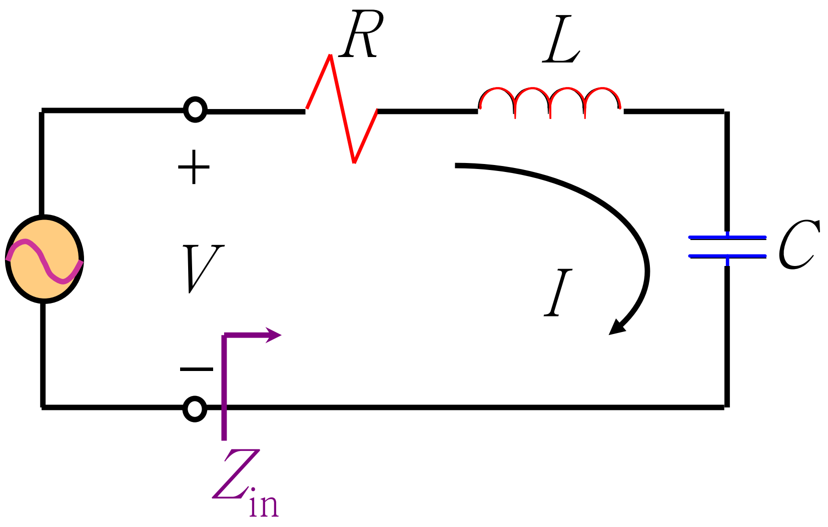
Resonant Circuits

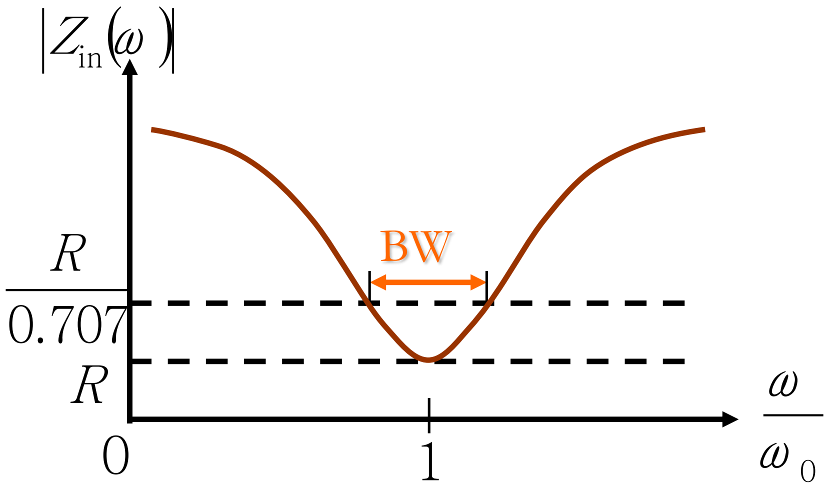
- Near resonance,
  - RF and microwave resonant circuits
- can be represented either as
  - a lumped element series or
  - parallel RLC networks.

Series Resonant Circuits

KNUST
Telecomm.
Engineering

Series Resonant Circuit





$$Z_{in} = R + j\omega L - j\frac{1}{\omega C}$$

Series Resonant Circuits

KNUST Telecomm. Engineering

- $$P_{in} = \frac{1}{2} V I^* = \frac{1}{2} Z_{in} I I^* = \frac{1}{2} Z_{in} |I|^2 = \frac{1}{2} V \left(\frac{V}{Z_{in}} \right)^* = \frac{1}{2} \frac{|V|^2}{Z_{in}^*}$$

$$= \frac{1}{2} |I|^2 \left(R + j\omega L - j \frac{1}{\omega C} \right) \qquad (6.2)$$

P_{loss} = Power dissipated by the resister $R \quad = \frac{1}{2} |I|^2 R$

W_m = Average magnetic energy stored in $L = \frac{1}{4} |I|^2 L$

W_e = Average electric energy stored in C

$$= \frac{1}{4} |V_C|^2 C = \frac{1}{4} \left| \frac{I}{j\omega C} \right|^2 C = \frac{1}{4} |I|^2 \frac{1}{\omega^2 C} \qquad \left(\because v_C = \frac{1}{C} \int i dt \qquad V_C = \frac{I}{j\omega C} \right)$$

Series Resonant Circuits

- $$P_{in} = P_{loss} + j2\omega(W_m - W_e) \qquad (6.5)$$

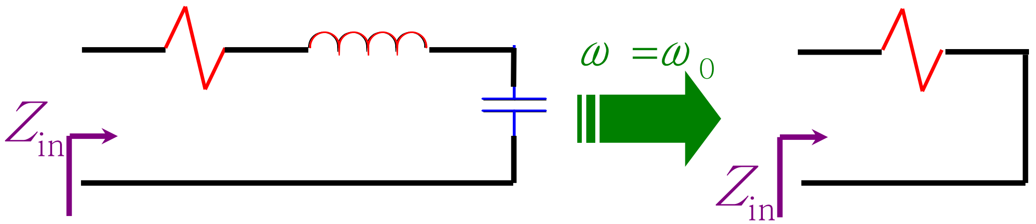
$$Z_{in} = \frac{2P_{in}}{|I|^2} = \frac{P_{loss} + j2\omega(W_m - W_e)}{|I|^2/2}$$

At resonance, $W_m = W_e$

$$\therefore\ Z_{in} = \frac{P_{loss}}{|I|^2/2} = R \quad \text{(purely real)}$$

$$W_m = \frac{1}{4}|I|^2 L = W_e = \frac{1}{4}|I|^2 \frac{1}{\omega^2 C}$$

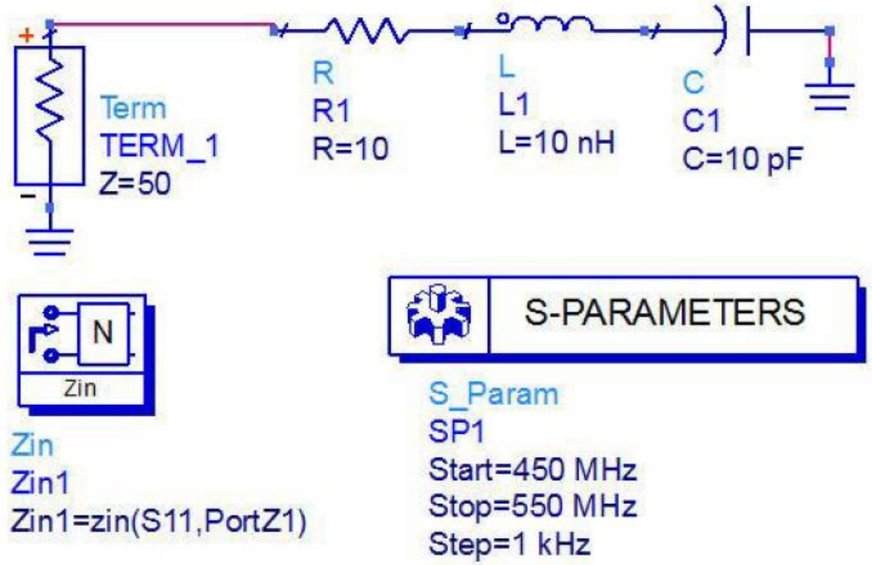
$$\therefore \omega = \frac{1}{\sqrt{LC}} = \omega_0 \quad \text{: resonant frequency}$$

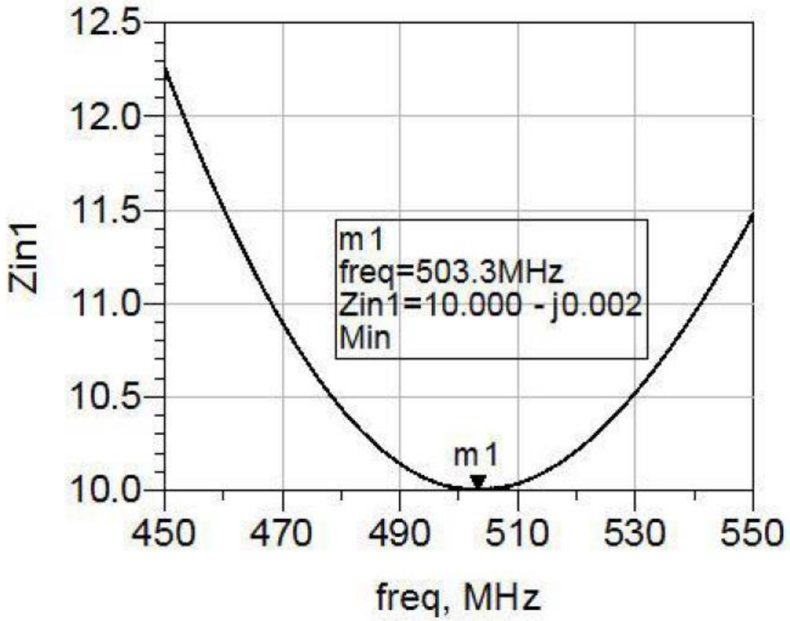


$$P_{in} = \frac{1}{2}|I|^2 R$$

Nea resonance, $Z_{in} = j2L\Delta\omega = j2L(\omega - \omega_0)$

Series Resonant Circuits





Series Resonant Circuits

Q (quality factor)

Quality Factor, Q of the series resonator is

$$Q=\omega\frac{(\text{average energy stored})}{(\text{energy loss/second})}=\omega\frac{W_m+W_e}{P_\ell}$$

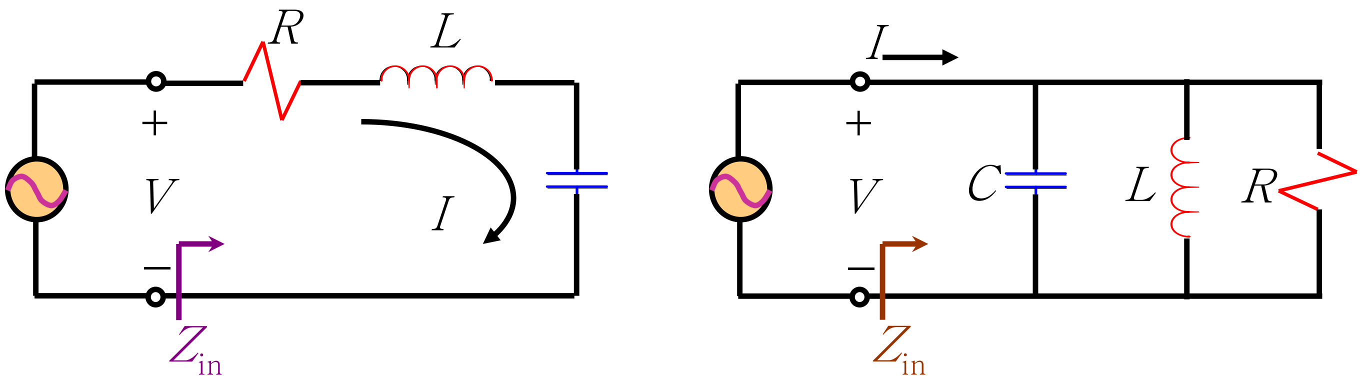
At resonance, $W_m=W_e$

$$\therefore\ Q=\omega_0\frac{2W_m}{P_\ell}=\omega_0\frac{2\frac{1}{4}|I|^2L}{\frac{1}{2}|I|^2R}=\frac{\omega_0L}{R}\left(=\frac{1}{\omega_0RC}\right)\ \left(\because\omega_0=\frac{1}{\sqrt{LC}}\right)$$

- As R decreases, Q increases

Parallel Resonant Circuits

Parallel Resonant Circuit



$$Z_{in} = R + j\omega L - j\frac{1}{\omega C} \quad \Rightarrow \quad Y_{in} = G + j\omega C - j\frac{1}{\omega L}$$

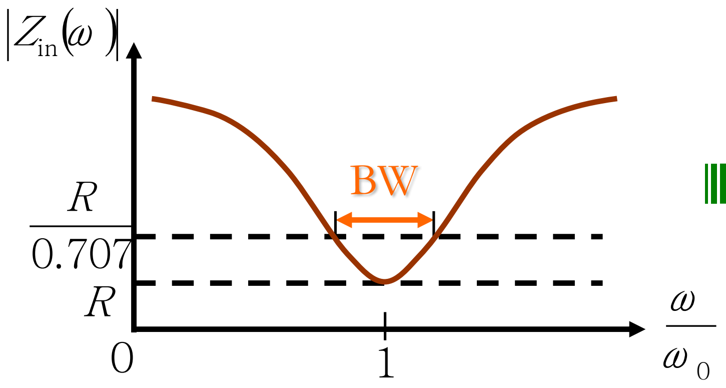
$$Q = \frac{\omega_0 L}{R} \quad \Rightarrow \quad Q = \frac{\omega_0 C}{G}$$

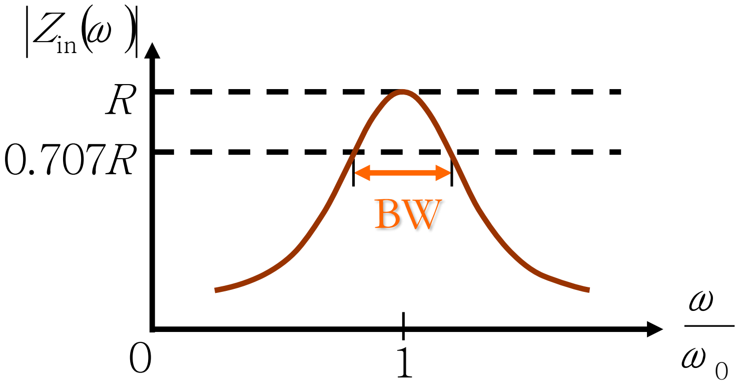
$$\omega_0 = \frac{1}{\sqrt{LC}} \quad \Rightarrow \quad \omega_0 = \frac{1}{\sqrt{LC}}$$

Parallel Resonant Circuits

$$\mathrm{BW} = \frac{1}{Q} \quad \Rightarrow \quad \mathrm{BW} = \frac{1}{Q}$$

$$Z_{\text{in}} \approx R + j2L\Delta\omega \quad (6.9) \quad \Rightarrow \quad Y_{\text{in}} \approx G + j2C\Delta\omega \quad (6.19)$$





Parallel Resonant Circuits

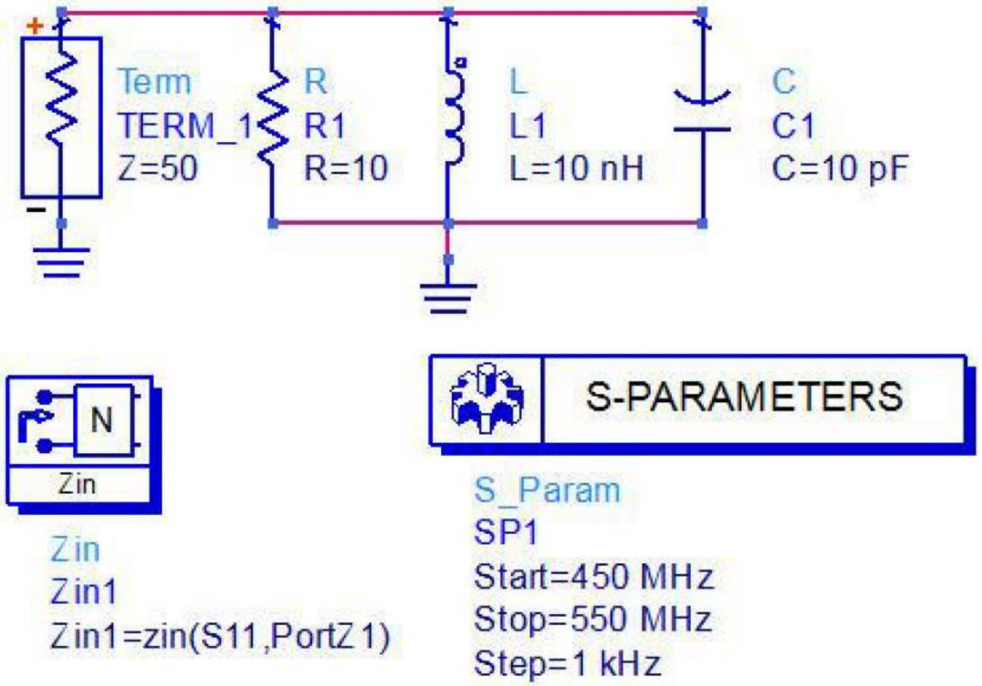
- If the AC voltage across the parallel resonant circuit is V,
  - then the complex power delivered to the resonator is

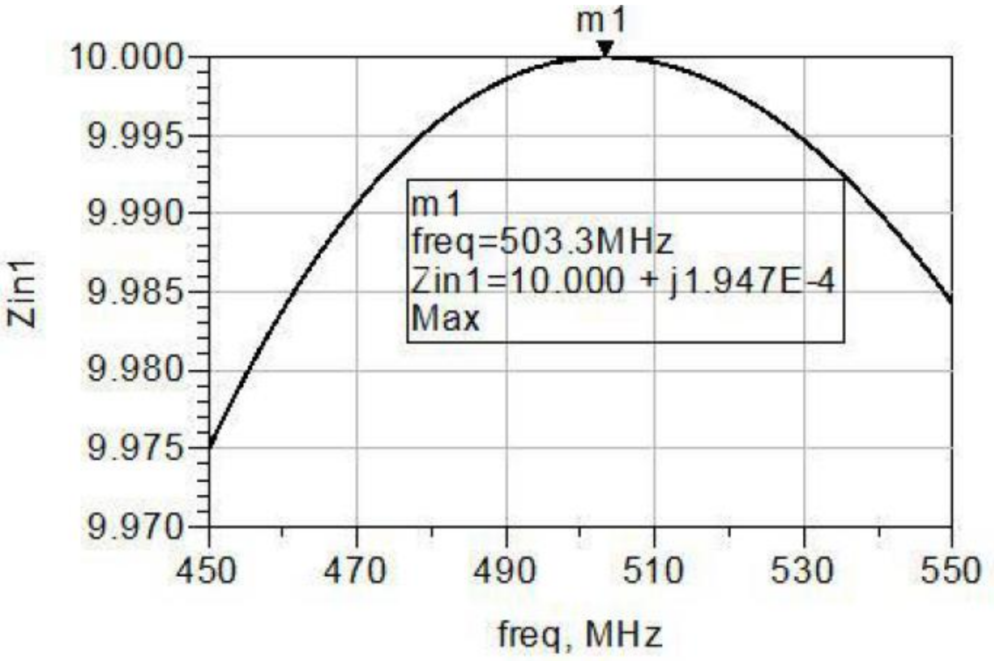
$$P_{in} = \frac{|V|^2}{2} Y_{in} = \frac{|V|^2}{2}\left(\frac{1}{R} + j\omega C - \frac{1}{j\omega L}\right)$$

- At resonance
  - power dissipated in the resistor is

$$P_{in} = \frac{|V|^2}{2R}, \qquad \omega_o = \frac{1}{\sqrt{LC}}$$

Parallel Resonant Circuits





Resonant Circuit Loss

- In the resonant circuits seen so far,
  - the resistor R_1 represents the loss in the resonator.
- It includes the losses in
  - the capacitor as well as
  - the inductor.
- The Q-factor is
  - the ratio of the energy stored in the inductor and capacitor to
  - the power dissipated in the resistor as a function of frequency

Resonant Circuit Loss

- For the series resonant circuit,
  - the unloaded *Q*-factor is defined by:

$$Q_u = \frac{X}{R} = \frac{\omega_o L}{R} = \frac{1}{\omega_o C R}$$

- The unloaded *Q*-factor of the parallel resonant circuit is simply
  - the inverse of the unloaded Q factor of the series resonant circuit.

$$Q_u = \frac{R}{X} = \frac{R}{\omega_o L} = \omega_o C R$$

Resonant Circuit Loss

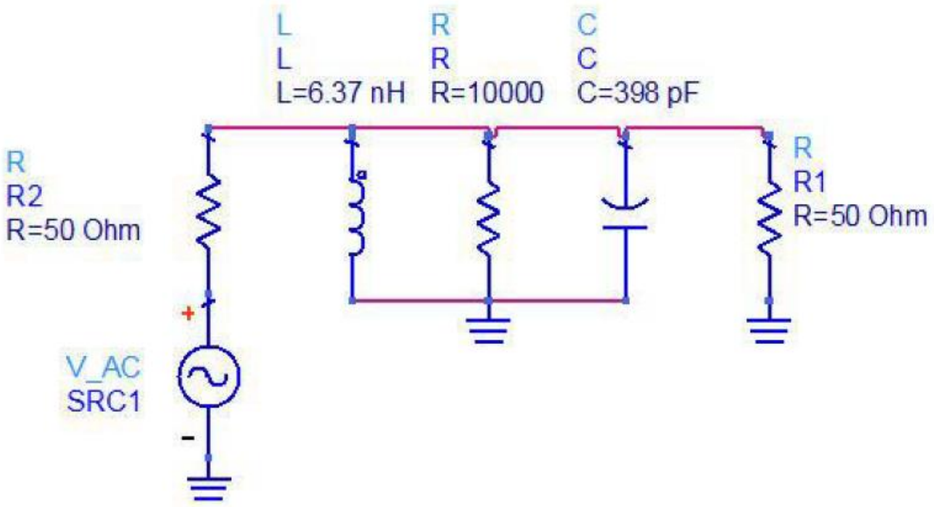
- We can clearly see that
  - as the resistance increases in the series resonant circuit,
  - the *Q*-factor decreases.
- Conversely,
  - as the resistance increases in the parallel resonant circuit,
  - the *Q*-factor increases.
- The *Q*-factor is a measure of loss in the resonant circuit. Thus
  - a higher Q corresponds to lower loss and
  - a lower Q corresponds to a higher loss.

Resonant Circuit Loss

KNUST
Telecomm.
Engineering

Loaded Q and External Q (Q_L and Q_e)

- For the parallel resonator that is attached to a 50Ω source and load as shown below



$$Q_L = \frac{R_S + R + R_L}{\omega_o L}$$

$$Q_E = \frac{R_S + R_L}{\omega_o L}$$

Resonant Circuit Loss

- The three Q-factors are related by the inverse relationship of equation

$$\frac{1}{Q_L} = \frac{1}{Q_u} + \frac{1}{Q_E}$$

- The loaded Q is also related to the fractional bandwidth as

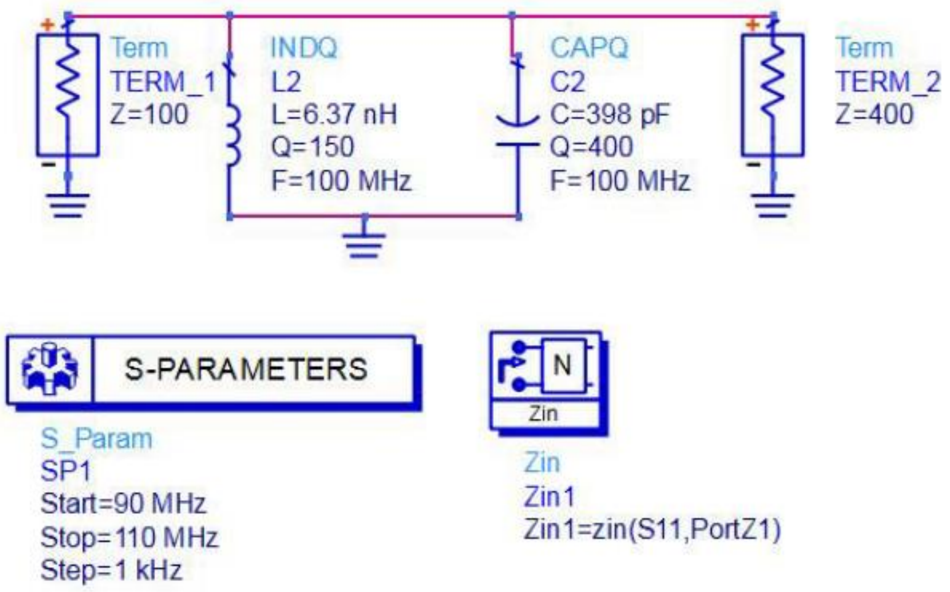
$$Q_L = \frac{\sqrt{f_l f_h}}{BW_{-3dB}}$$

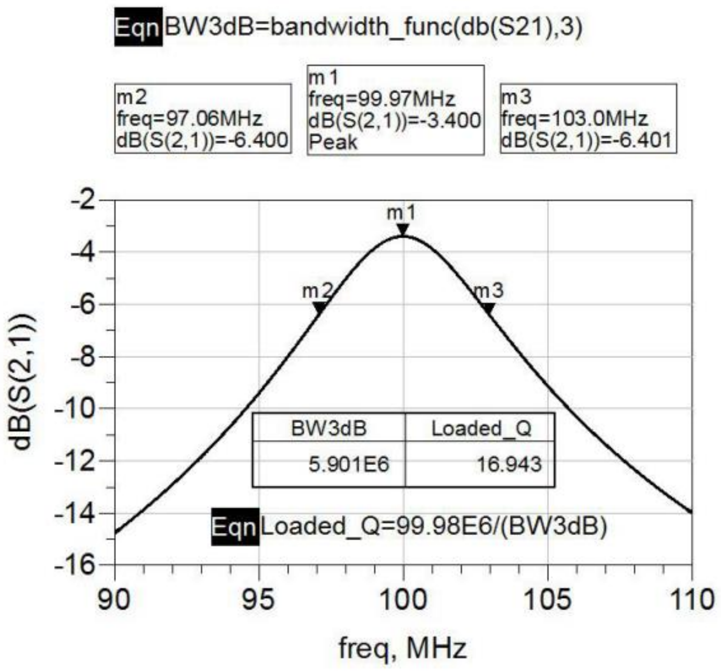
  - BW is the -3dB bandwidth,
  - f_l and f_h are the lower and upper frequencies in Hz at -3dB points.

Resonant Circuit Loss

- Example
- Design a lumped element parallel resonator at a frequency of 100 MHz. The resonator is intended to operate between a source resistance of 100 Ω and a load resistance of 400 Ω. use the ADS inductor and capacitor models that include the comp onent Q factor (L=6.37 nH, Q =150; C=398 pF, Q=400).

Resonant Circuit Loss





The loaded Q can be calculated from

$$Q_L = \frac{\sqrt{f_l f_h}}{BW_{-3dB}} = \frac{99.98MHz}{5.901MHz} = 16.4$$

Resonant Circuit Loss

Effect of Load Resistance on BW and Q_L

- In RF circuits and systems
  - the impedances encountered are often quite low,
  - ranging from 1 Ω to 50 Ω.
  - It may not be practical to have a source impedance of 100 Ω and a load impedance of 400 Ω.
- Lower resistances such as 50 Ω will
  - Lower the bandwidth and thus
  - Lower the loaded Q

Resonant Circuit Loss

Effect of Load Resistance on BW and Q_L

- To maintain the high Q of the resonator when attached to a load such as 50 Ω,
  - it is necessary to transform the low impedance to high impedance presented to the load.
  - The 50 Ω impedance can be transformed to the higher impedance of the parallel resonator thereby
    - resulting in less loading of the resonator impedance.
  - This is referred to as loosely coupling the resonator to the load.

Resonant Circuit Loss

- The tapped-capacitor and tapped-inductor networks
  - can be used to accomplish this *Q* transformation in lumped element circuits.
- Replacing the capacitor in the parallel network with a tapped capacitor,

$$C_T = \frac{C_1 C_2}{C_1 + C_2}$$

$$R_{L1} = R_L \left(1 + \frac{C_1}{C_2}\right)^2$$

  - R_{L1} is the higher transformed load resistance (required for the high Q)
  - R_L is the desired termination impedance, say 50 Ω
  - C_T is simply the original capacitance and C1 and C2 results from tapping

Resonant Circuit Loss

- A tapped inductor can also be used to the same effect at the source

$$R_{S1} = R_S \left(\frac{L_T}{L_1} \right)^2$$

$$L_T = L_1 + L_2$$

Practical Microwave Resonators

- At higher RF and microwave frequencies
  - small values of inductance and capacitance are physically unrealizable.
  - So resonators are seldom realized with discrete lumped element RLC components.
  - Even if the values could be physically realized the resulting Q factors would be unacceptably low for most applications.

Practical Microwave Resonators

- At higher RF and microwave frequencies
  - Resonators can be realized in all of the basic transmission line forms
  - There are many specialized resonators such as
    - ceramic dielectric resonator pucks that are coupled to a microstrip transmission line as well as
    - Yittrium Iron Garnet spheres that are loop coupled to its load.
  - These resonators are optimized for very high Q factors
  - and may be tunable over a range of frequencies.

Resonant Circuit Loss

Home Work

- In the parallel LC Example, change the load from 400 Ω to 50 Ω and re-examine the circuit's 3 dB bandwidth and Q_L .
- Rearrange the parallel LC network with the tapped capacitor network and Re-examine the circuit's 3 dB bandwidth and Q_L.
- Do same for the inductor and re-examine the circuit's 3 dB bandwidt h and Q_L.