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TE 364

LECTURE 6

Filter Circuits

2018.02.25

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ICnSYS LAB

Outline

- Basic Filters and Terminology
  - Low-pass, High-pass, band-pass and band-stop
- Passive Filters Synthesis
  - Image Parameter Method
  - Insertion Loss Method
- Microwave Filters

Introduction

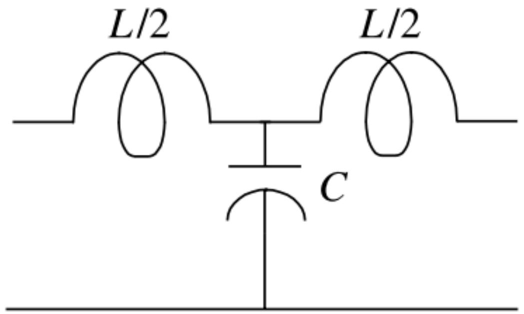
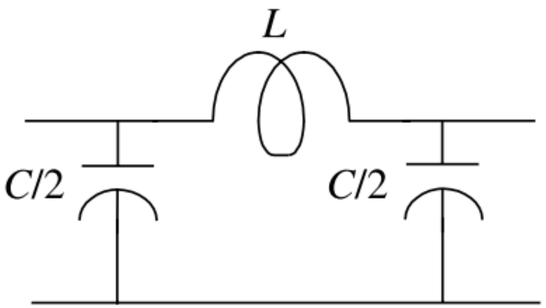
A Filter

- is a two-port network
- used to control the frequency response at a certain point in an RF or microwave system
- by providing transmission at frequencies within the passband of the filter and
- attenuation in the stopband of the filter.

Basic Filter Types

- Typical frequency responses include:
  - low-pass,
  - high-pass,
  - bandpass, and
  - band-reject characteristics.

Basic Filter Types

<table>
<tr><th>Filter Type</th><th>T-Section</th><th>π-Section</th></tr>
<tr><td>Low-pass</td><td>
</td><td>
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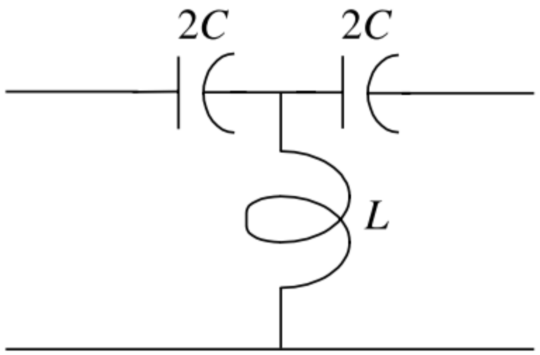
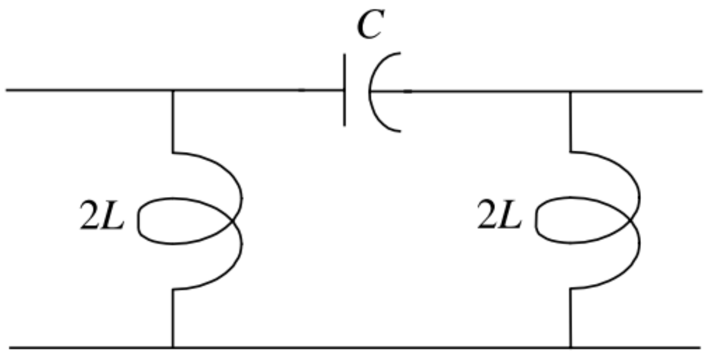
<tr><td>High-pass</td><td>
</td><td>
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Basic Filter Types

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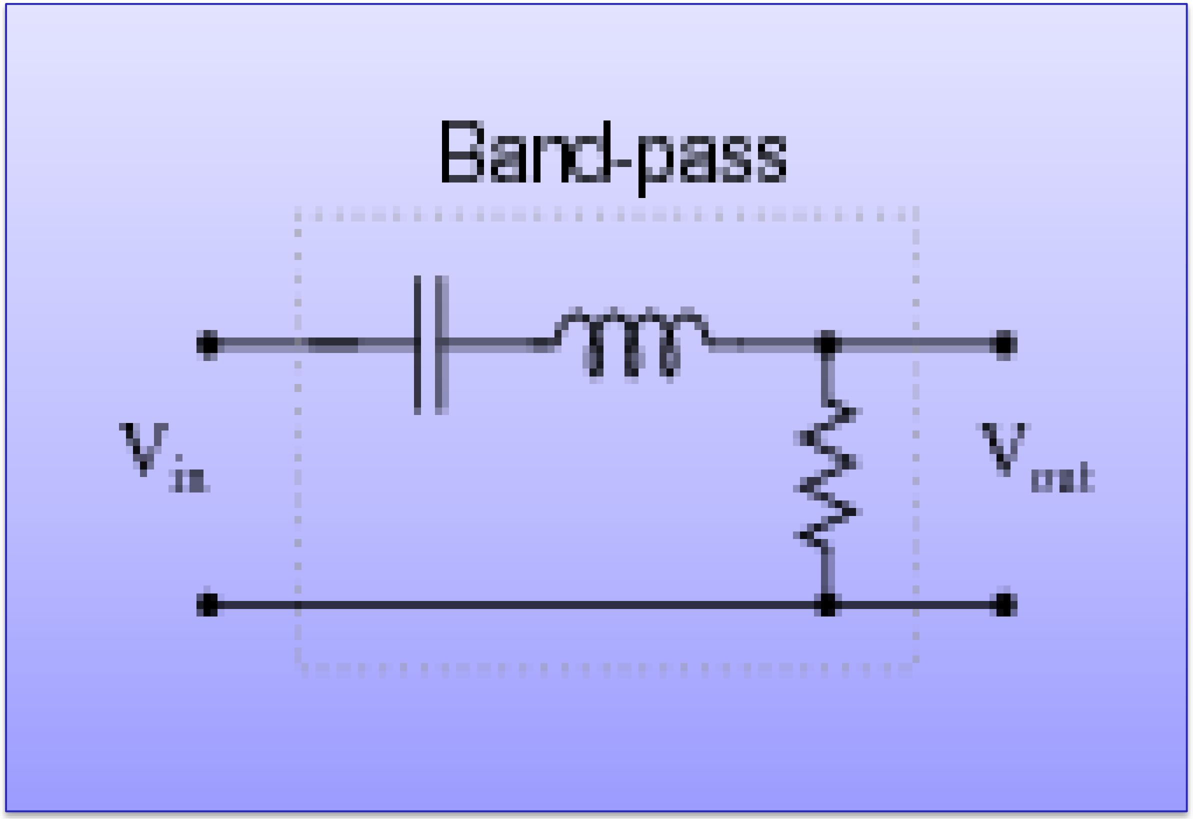

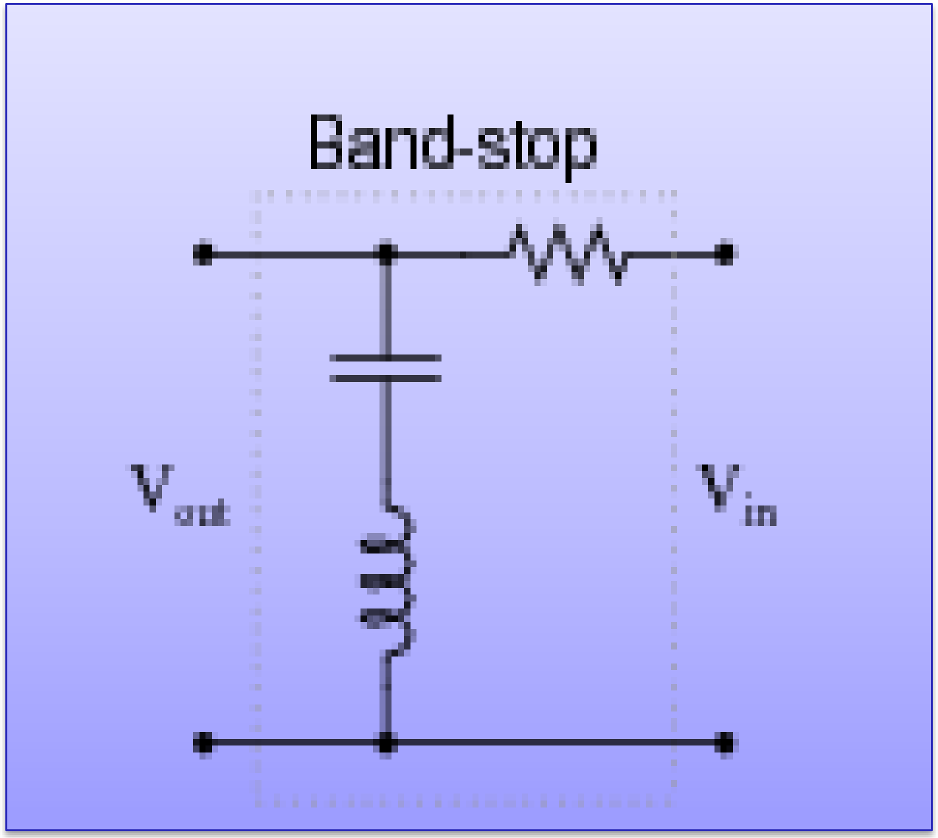

Filter Frequency Response

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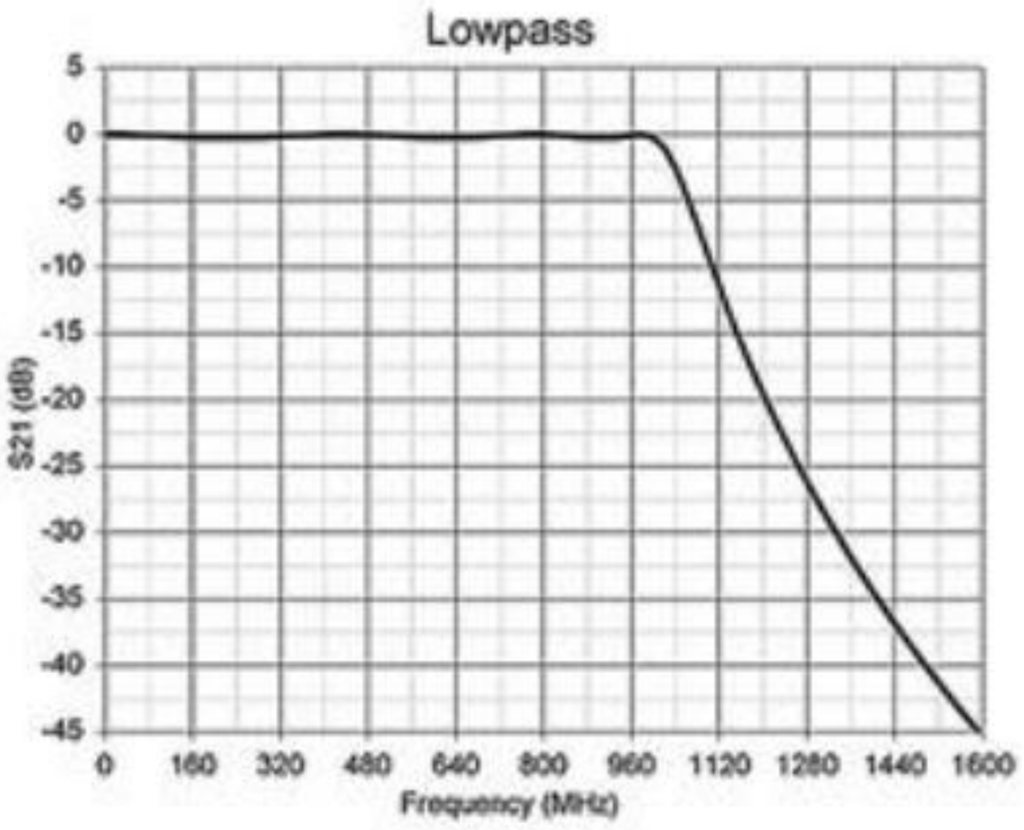

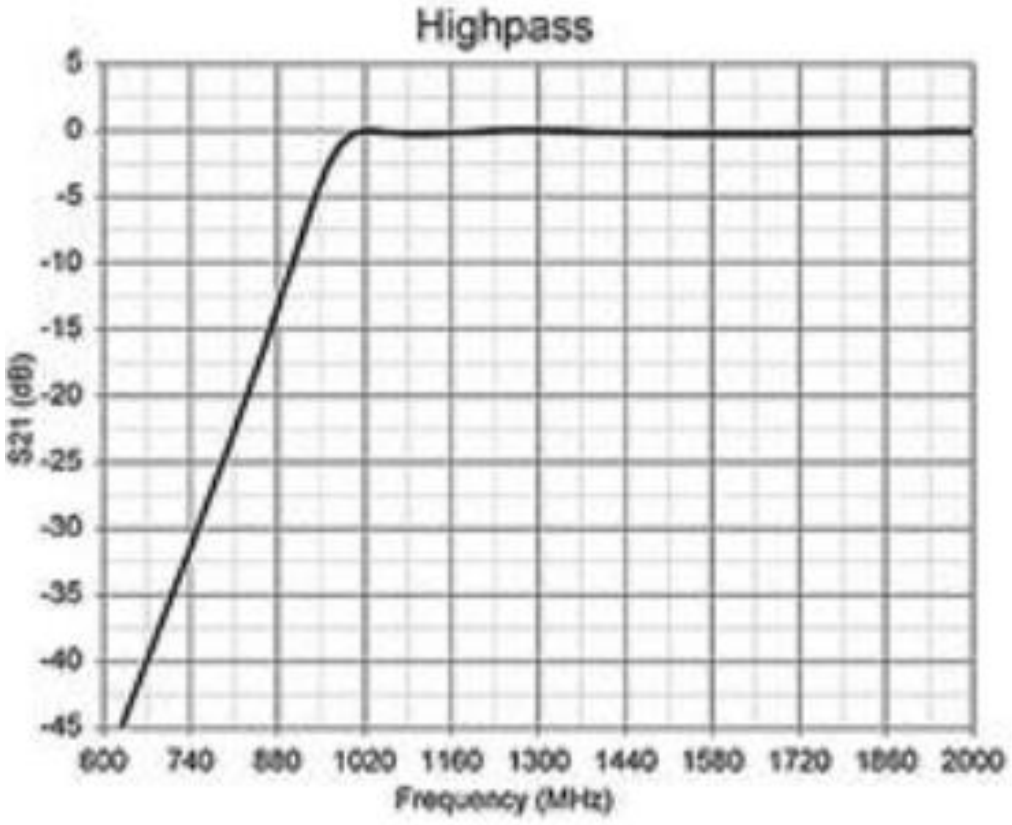

Filter Frequency Response

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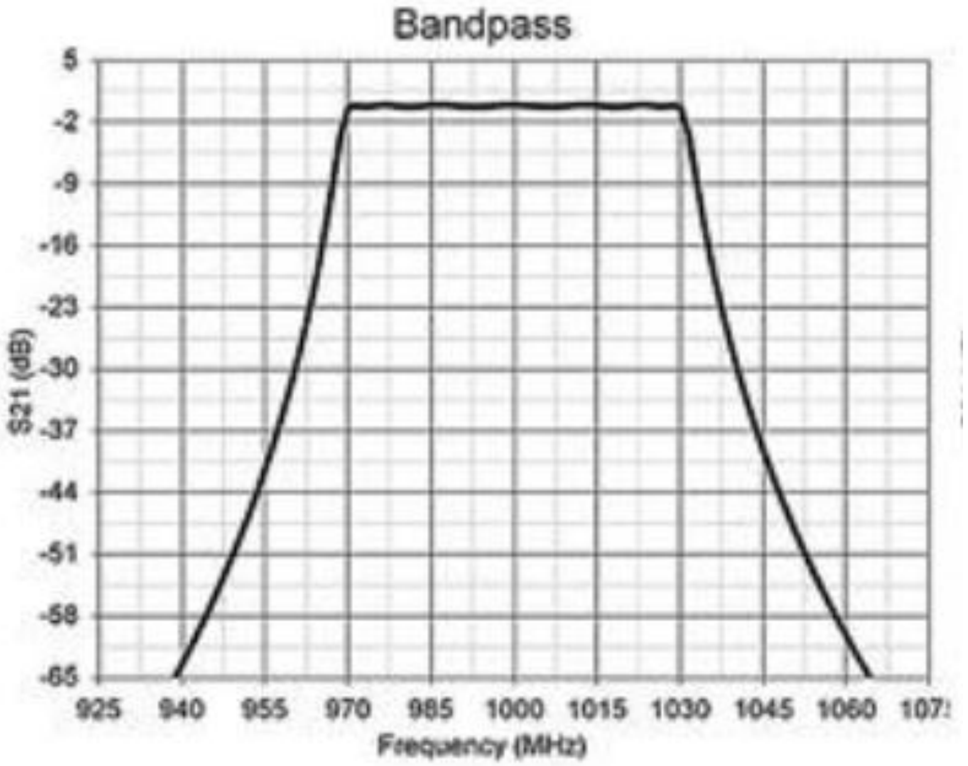

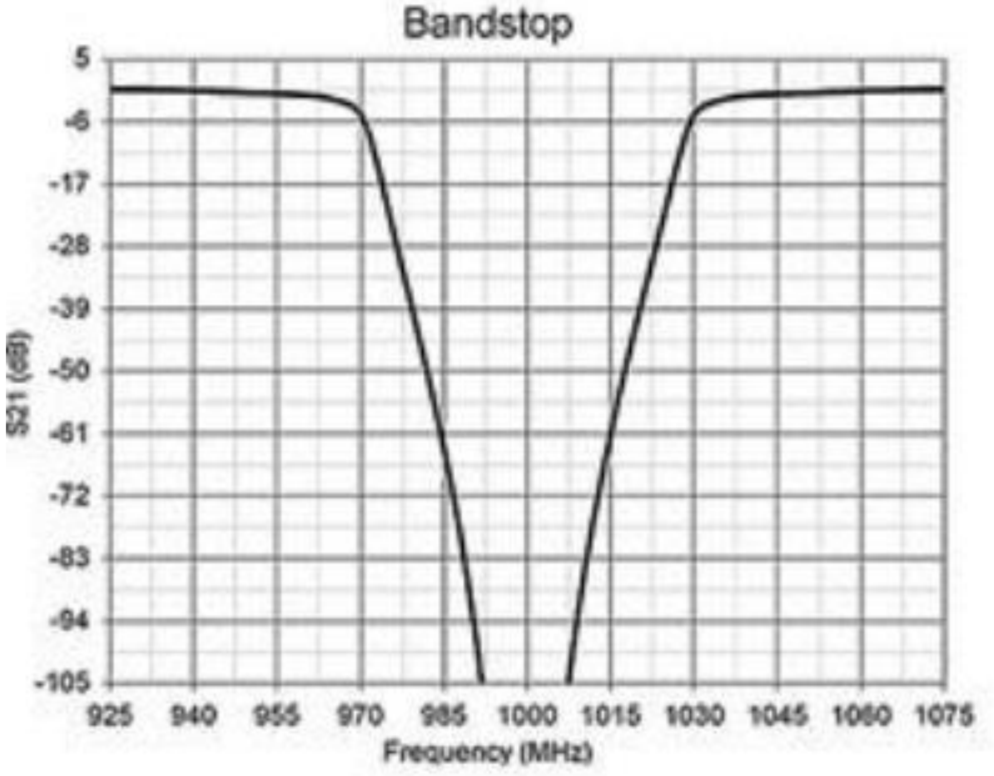

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Important Terminologies

- Insertion Loss (in dB)
  - The ratio of the power delivered by a source to a load with and without a two-port network inserted in between.
- Return Loss (in dB)
  - The fraction of the input power that is lost due to reflection at its in put port
- Attenuation (in dB or Nepers)
  - The ratio of the power delivered to a matched load to that supplied to it by a matched source

Filter Classification

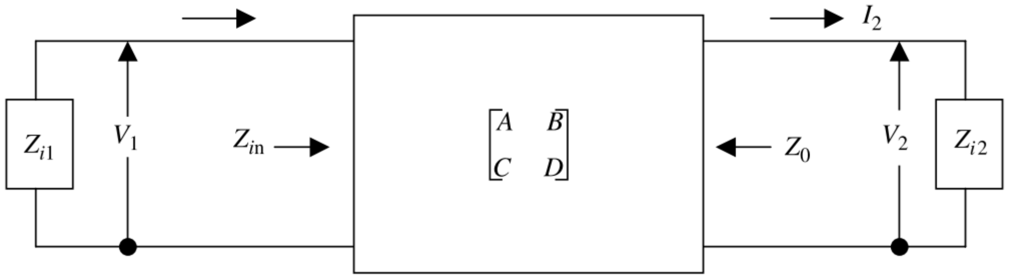
- Active filters
  - can amplify the signal besides blocking the undesired frequencies
- Passive filters
  - are economical and easy to design.
  - perform fairly well at higher frequencies

Filter Design Methods

- *Image parameter method*
  - provides a design that can pass or stop a certain frequency band but
  - its frequency response cannot be shaped.
- *Insertion-loss method*
  - Is more powerful in the sense that
  - it provides a specified response of the filter.

Image Parameter Method

- Consider the two-port network



  - Z_{i1} and Z_{i2} are the image impedance of the network

$$V_1 = AV_2 + BI_2$$
$$I_1 = CV_2 + DI_2$$

$$Z_{in} = \frac{V_1}{I_1} = \frac{AV_2 + BI_2}{CV_2 + DI_2} = \frac{AZ_{i2} + B}{CZ_{i2} + D}$$

Image Parameter Method

- It can be shown that

$$V_2 = DV_1 - BI_1$$
$$I_2 = -CV_1 + AI_1$$

$$Z_o = -\frac{V_2}{I_2} = -\frac{DV_1 - BI_1}{-CV_1 + AI_1} = \frac{DZ_{i1} + B}{CZ_{i1} + A}$$

- *Note that*

$$Z_{i1} = -\frac{V_1}{I_1}$$

Image Parameter Method

- It can further be shown that;
  - For $Z_{i1} = Z_{in}$ and $Z_{i2} = Z_o$

$$Z_{i1} = \sqrt{\frac{AB}{CD}}$$

  - and

$$Z_{i2} = \sqrt{\frac{BD}{AC}}$$

Image Parameter Method

- Furthermore;
  - The transfer characteristics is given by

$$\frac{V_1}{V_2} = A + B\frac{I_2}{V_2} = A + \frac{B}{Z_{i2}}$$

  - or

$$\frac{V_1}{V_2} = \sqrt{\frac{A}{D}}\left(\sqrt{AD} + \sqrt{BC}\right)$$

Image Parameter Method

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- Or

$$\frac{V_2}{V_1} = \sqrt{\frac{D}{A}}\left(\sqrt{AD} - \sqrt{BC}\right)$$

- *Note that,* $AD - BC = 1$ *for reciprocal networks*

- Similarly

$$\frac{I_2}{I_1} = \sqrt{\frac{A}{D}}\left(\sqrt{AD} - \sqrt{BC}\right)$$

A/D is X'former ratio taken as 1 in this case

- And $e^{-\gamma} = \sqrt{AD} - \sqrt{BC}$ or $\cosh\gamma = \sqrt{AD}$

Image Parameter Method

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❖Parameters for T and Pi-networks

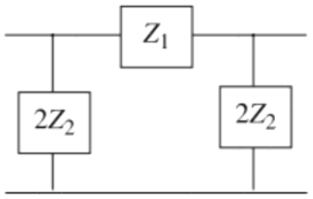
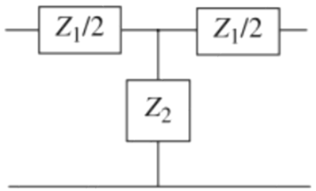
	π-Network	T-Network
		
ABCD parameters	$A = 1 + \frac{Z_1}{2Z_2}$	$A = 1 + \frac{Z_1}{2Z_2}$
	$B = Z_1$	$B = Z_1 + \frac{Z_1^2}{4Z_2}$
	$C = \frac{1}{Z_2} + \frac{Z_1}{4Z_2^2}$	$C = \frac{1}{Z_2}$
	$D = 1 + \frac{Z_1}{2Z_2}$	$D = 1 + \frac{Z_1}{2Z_2}$
Image impedance	$Z_{i\pi} = \sqrt{\frac{Z_1 Z_2}{1 + Z_1/4Z_2}} = \frac{Z_1 Z_2}{Z_{iT}}$	$Z_{iT} = \sqrt{Z_1 Z_2 \left(1 + \frac{Z_1}{4Z_2}\right)}$
Propagation constant, γ	$\cosh\gamma = 1 + \frac{Z_1}{2Z_2}$	$\cosh\gamma = 1 + \frac{Z_1}{2Z_2}$

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Image Parameter Method

❖ Constant *k*-filter sections

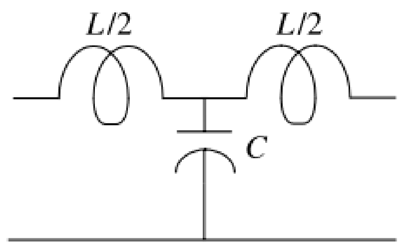
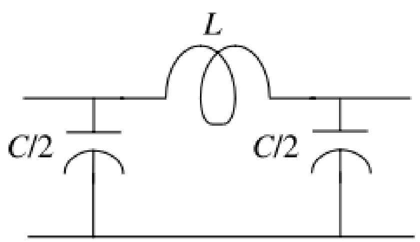
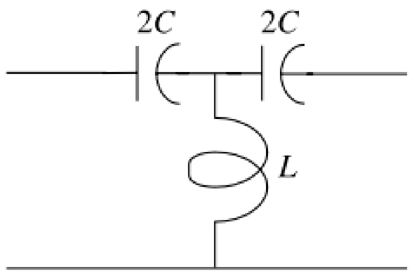
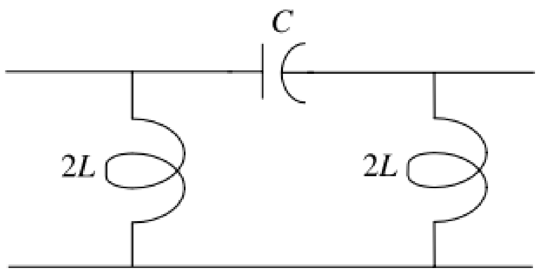
Filter Type	T-Section	π-Section
Low-pass		
High-pass		

Image Parameter Method

- For the low-pass T-section,

$$Z_1 = j\omega L \qquad Z_2 = \frac{1}{j\omega C}$$

- Therefore, the image impedance from Table is

$$Z_{iT} = \sqrt{\frac{L}{C}\left(1 - \frac{\omega^2 LC}{4}\right)}$$

- In the case of D.C.

$$Z_{iT} = \sqrt{\frac{L}{C}}$$ *(Nominal Impedance)*

Image Parameter Method

- For Z_{iT} to be equal to 0, (*Cut-off frequency, ω_c*)

- $$\frac{\omega^2 LC}{4} = 1 \Rightarrow \omega_c = \frac{2}{\sqrt{LC}}$$

Image Parameter Method

- For the high-pass T-section,

$$Z_1 = \frac{1}{j\omega C} \qquad Z_2 = j\omega L$$

  - Therefore, the image impedance from Table is

$$Z_{iT} = \sqrt{\frac{L}{C}\left(1 - \frac{1}{4\omega^2 LC}\right)}$$

  - The cut-off frequency is

$$\omega_c = \frac{1}{2\sqrt{LC}}$$

Image Parameter Method

- ***Example***
  - Design a low-pass constant-k T-section that has a nominal impedance of 75 Ω and a cut-off frequency of 2 MHz.

- ***Solution***

$$Z_{iT} = 75\Omega = \sqrt{\frac{L}{C}} \ , \qquad \omega_c = 2\times 2\pi \times 10^6 = \frac{2}{\sqrt{LC}}$$

$$L = 11.9366\ \mu\text{H and } \ C = 2.122\ n\text{F}$$

Image Parameter Method

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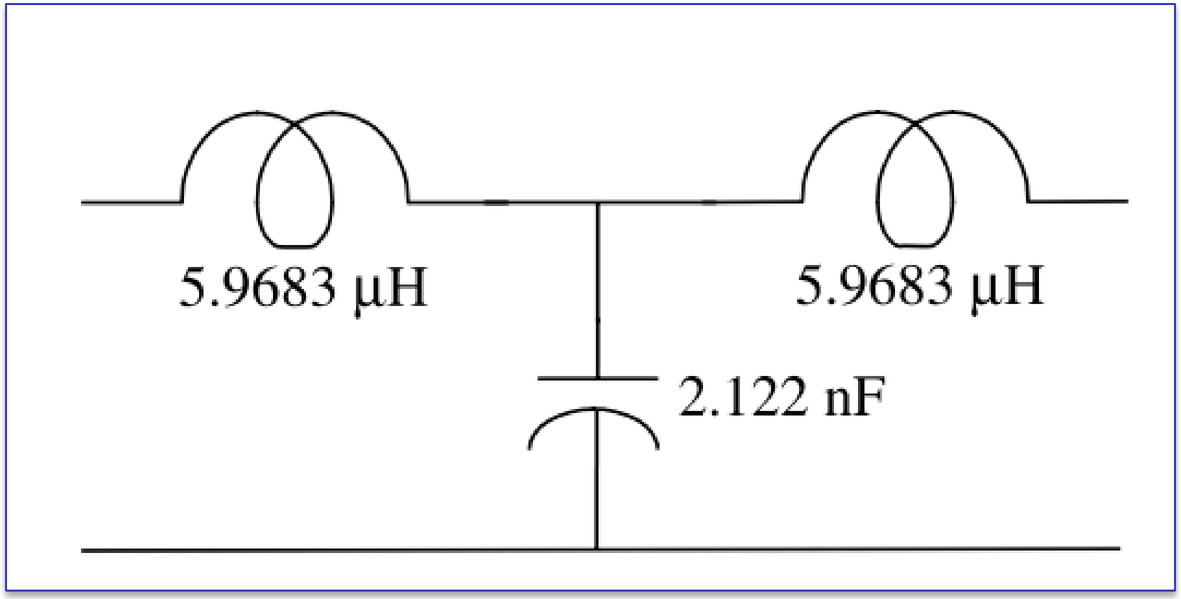

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Image Parameter Method

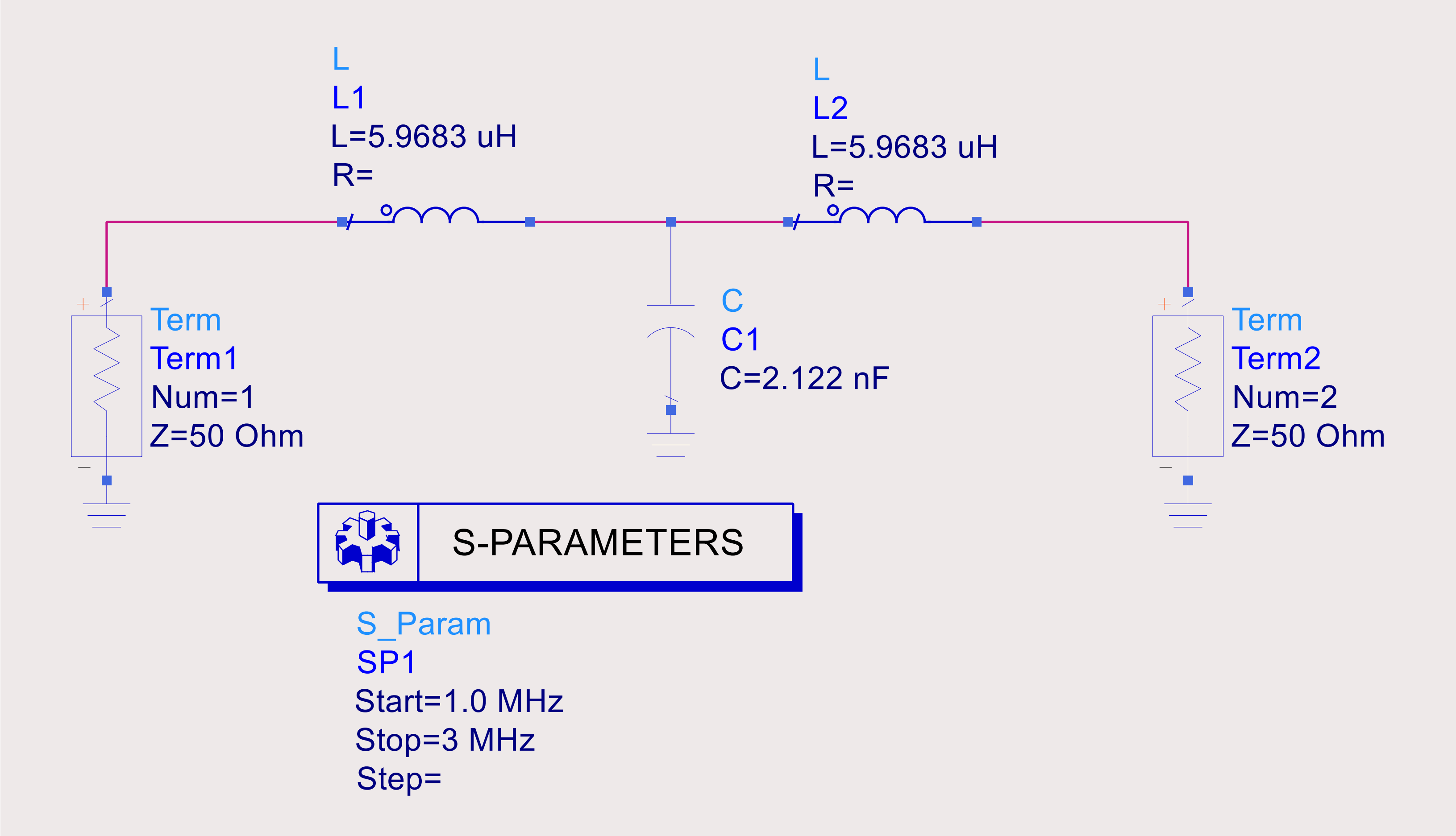


Image Parameter Method

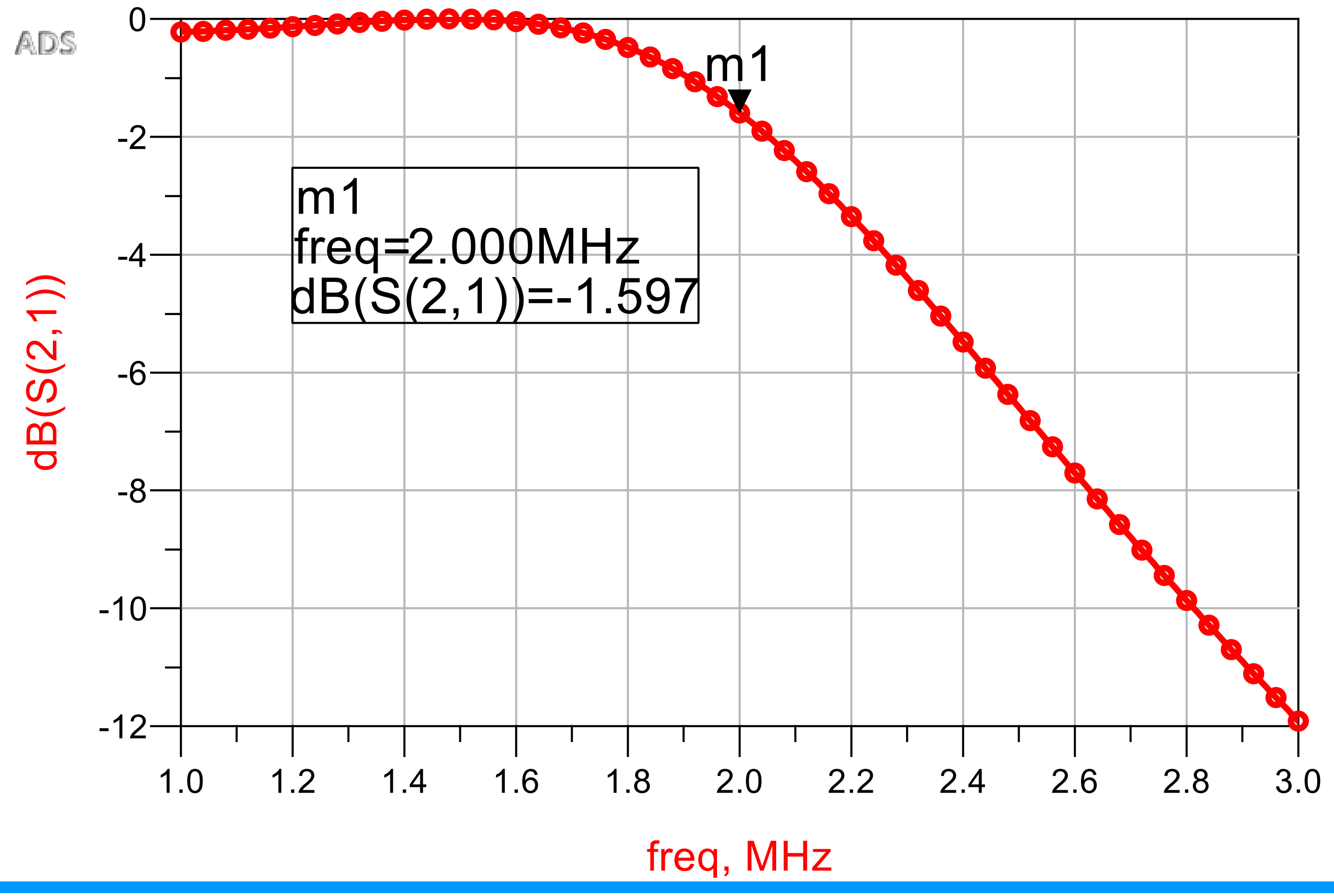
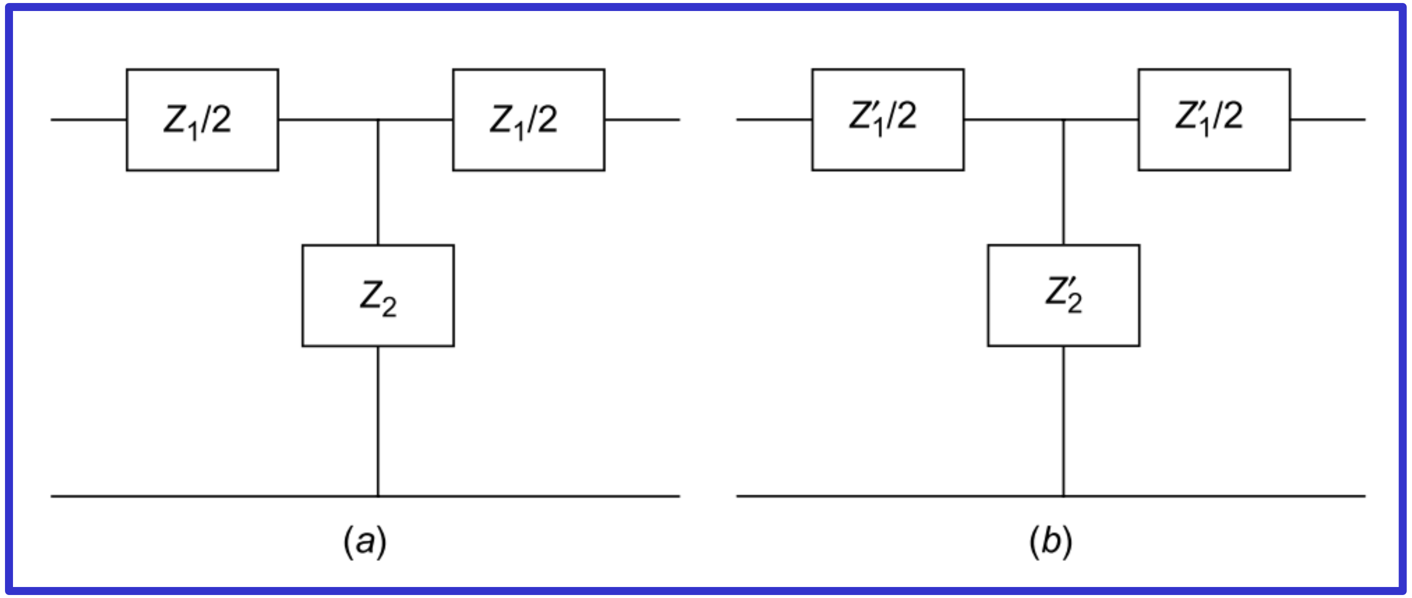


Image Parameter Method

- **Disadvantage of Image Parameter**
  - The signal attenuation rate after the cut-off point is not very sharp,
  - The image impedance is not constant with frequency.
  - From a design point of view, it is important that it is constant, at least in its pass-band.
- Solution:
  - **M-derived Filter Section**

M-Derived Filter Section

- Consider the following two T-sections



$$Z_{iT} = Z'_{iT} = \sqrt{Z_1' Z_2' \left(1 + \frac{Z_1'}{4Z_2'}\right)} = \sqrt{Z_1 Z_2 \left(1 + \frac{Z_1}{4Z_2}\right)}$$

M-Derived Filter Section

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$$Z_2' = \frac{1}{Z_1'}\left(Z_1 Z_2 + \frac{Z_1^2 - Z_1'^2}{4} \right)$$

Let $Z_1' = mZ_1$

$$Z_2' = \frac{Z_2}{m} + \frac{1-m^2}{4m} Z_1$$

- Thus, an *m*-derived section is designed from the values of components determined for the corresponding constant-*k* filter.
- The value of m is selected to sharpen the attenuation at cut-off
- or to control the image impedance characteristics in the pass-band.

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M-Derived Filter Section

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- For the low-pass T-section,

$$Z_1' = j\omega mL \qquad Z_2' = \frac{1-m^2}{4m} j\omega L + \frac{1}{j\omega mC}$$

- Therefore, the transfer function is

$$\frac{Z_1'}{Z_2'} = -\frac{\omega^2 m^2 LC}{1-\left[\left(1-m^2\right)/4\right]\omega^2 LC} = \frac{4\omega^2 m^2/\omega_c^2}{1-\left(1-m^2\right)\omega^2/\omega_c^2}$$

$$\omega_c = \frac{2}{\sqrt{LC}}$$

M-Derived Filter Section

- From the table of parameters on slide 17,

$$\cosh\gamma = 1 + \frac{Z_1'}{2Z_2'} = 1 - \frac{2\left(m\omega/\omega_c\right)^2}{1-\left(1-m^2\right)\left(\omega/\omega_c\right)^2}$$

$$\cosh\gamma = \frac{\omega_c^2 - \omega^2 - \left(m\omega\right)^2}{\omega_c^2 - \left(1-m^2\right)\omega^2}$$

M-Derived Filter Section

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- $\cosh\gamma \to \infty$ if

$$\omega = \frac{\omega_c}{\sqrt{1-m^2}} = \omega_\infty$$

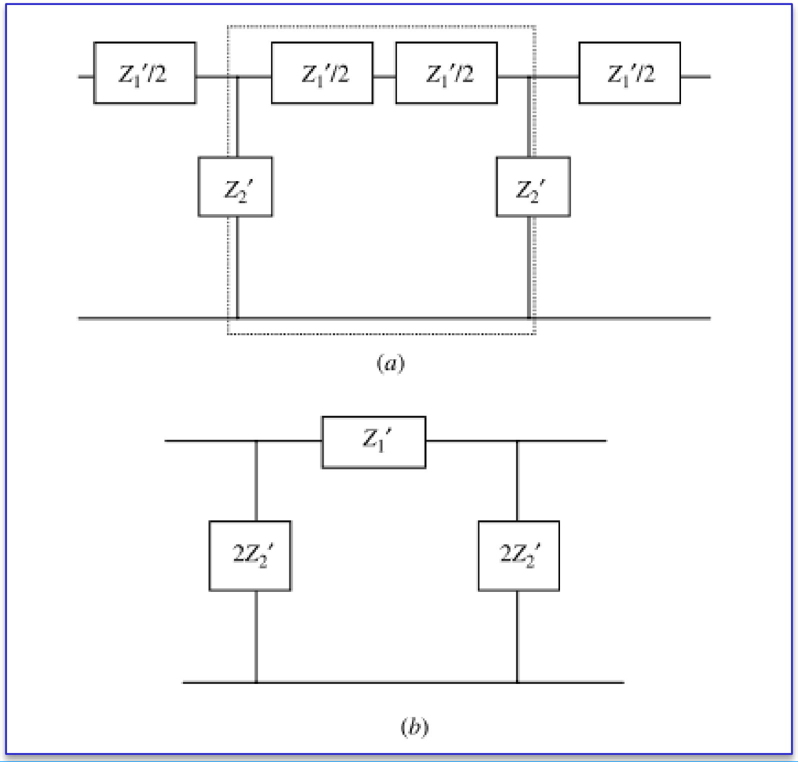
- Condition used to sharpen *attenuation cut-off*
  - Small m means $\omega \cong \omega_\infty$
  - ω_∞ selected slightly higher than ω_c
  - m is then determined from the above condition
  - Z_1' and Z_2' can then be determined from equations on slide 21

M-Derived Filter Section

- Note: image impedance of *m*-derived T-section
  - is same as that of corresponding constant-*k* network
- In the case of Pi-network
  - It is a function of *m*
- A characteristics used to design input and output network of the filter so that:
  - Image impedance of composite network stays constant in its pass band

M-Derived Filter Section

- Note: an infinite cascade of T-networks
  - Can be considered as Pi-network after splitting shunt arm.



M-Derived Filter Section

- Note: of th Z_2' Γ-network is replaced by $2Z_2'$
- Also two halves of the series arms give Z_1'
- From the table of parameters on slide 13,

$$Z_{i\pi} = \frac{Z_1'Z_2'}{Z_{iT}} = \frac{Z_1Z_2 + \left[\left(1-m^2\right)/4\right]Z_1^2}{Z_{iT}}$$

- For the low-pass constant k-filter

$$Z_1Z_2 = \frac{L}{C} = Z_o^2 \qquad Z_1^2 = -\omega^2 L = -\left(\frac{2Z_o\omega}{\omega_c}\right)^2$$

M-Derived Filter Section

and,

$$Z_{iT} = Z_0 \sqrt{1 - \left(\frac{\omega}{\omega_c}\right)^2}$$

Therefore,

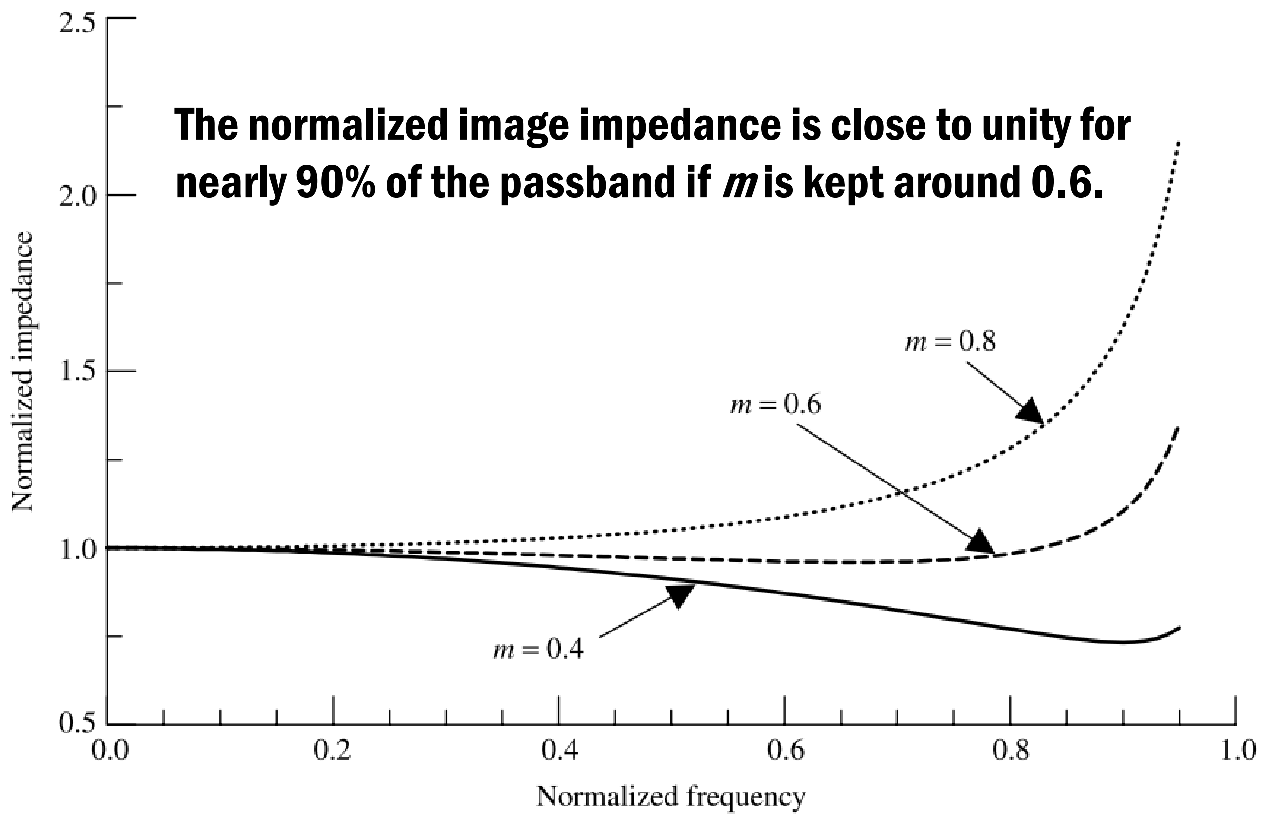
$$Z_{i\pi} = \frac{1 - (1 - m^2)(\omega/\omega_c)^2}{\sqrt{1 - (\omega/\omega_c)^2}} Z_0$$

$$or \;\; \overline{Z}_{i\pi} = \frac{1 - \left(1 - m^2\right)\overline{\omega}^2}{\sqrt{1 - \overline{\omega}^2}}$$

$$\overline{Z}_{i\pi} = Z_{i\pi} / Z_o$$

$$\overline{\omega} = \omega / \omega_c$$

M-Derived Filter Section



Normalized image impedance of π -network versus normalized frequency for three values of *m*.

M-Derived Filter Section

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❖ ***Example***

- Design an *m*-derived T-section low pass filter with a cut -off frequency of 2 MHz and a nominal impedance of 75 Ω. The infinity frequency f_∞ is 2.05 MHz.

❖ ***Solution***

$$Z_{iT} = 75\Omega = \sqrt{\frac{L}{C}} \qquad \omega_c = 2\times\pi\times10^6 = \frac{2}{\sqrt{LC}}$$

$$L = 11.9366\ \mu\text{H and } C = 2.122\ n\text{F}$$

$$\omega = \frac{\omega_c}{\sqrt{1-m^2}} = \omega_\infty \Rightarrow 1-m^2 = \left(\frac{f_c}{f_\infty}\right)^2$$

M-Derived Filter Section

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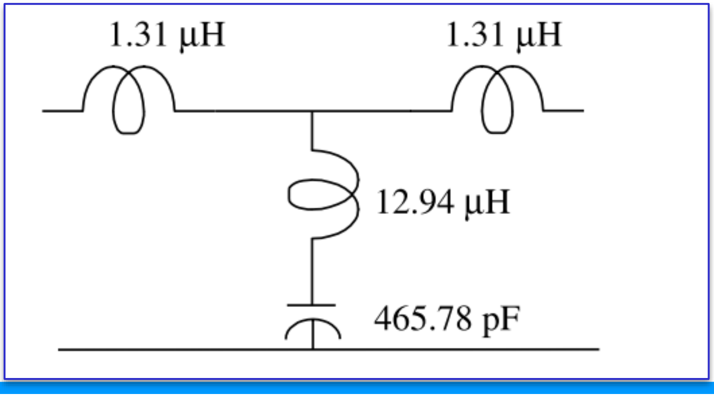
$$m = \sqrt{1 - \left(\frac{f_c}{f_\infty}\right)^2} = \sqrt{1 - \left(\frac{2}{2.05}\right)^2} = 0.2195$$

$$mL/2 = 0.2195 \times 5.9683 = 1.31\ \mu\text{H}$$

$$mC = 0.2195 \times 2.122 = 465.78\ \text{nF}$$

$$\frac{1 - m^2}{4m} L = 12.94\ \mu\text{H}$$



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Composite Filters

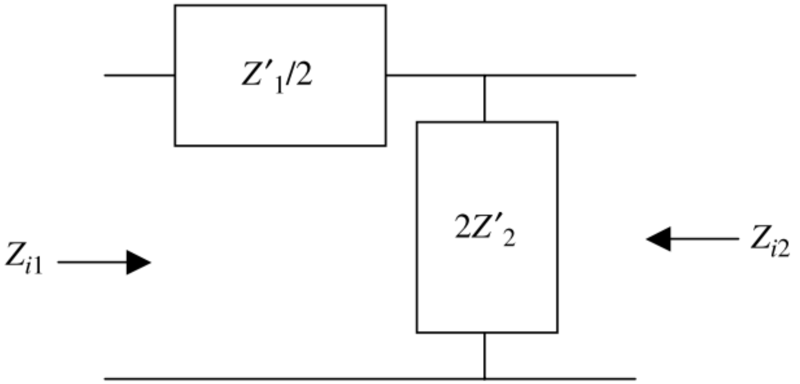
- *m*-derived filter provides sharp attenuation at cut-off
  - but attenuation in its stop-band is unacceptably low
- constant-*k* filter shows a higher attenuation in its stop-band,
  - but the change is unacceptably gradual
- Composite Filters: cascading these two filters
  - Taking advantage of each.

Composite Filters

- Since image impedance stays the same in two cases,
  - this cascading will not create a new impedance matching problem.
  - However image impedance varies with frequency
    at the input and output ports of the network.
- Input and output matching therefore required

Composite Filters

- Consider the bisected pi-section below



$$A = 1 + \frac{Z_1'}{4Z_2'} \qquad B = \frac{Z_1'}{2} \qquad C = \frac{1}{2Z_2'} \qquad D = 1$$

Composite Filters

- The image impedances are:

$$Z_{i1} = \sqrt{Z_1' Z_2' + \frac{Z_1'^2}{4}} = Z_{iT}$$

$$Z_{i2} = \sqrt{\frac{Z_1' Z_2'}{1 + Z_1'/4Z_2'}} = \frac{Z_1' Z_2'}{Z_{iT}} = Z_{i\pi}$$

  - the bisected π -section can be connected at the input and output ports of cascaded constant-k and *m*-deri-ved sections to obtain a composite filter that

- solves the impedance problem.

M-Derived Filter Section

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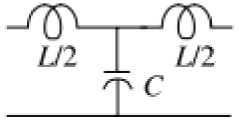
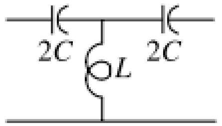
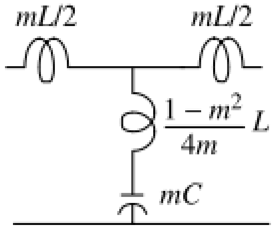
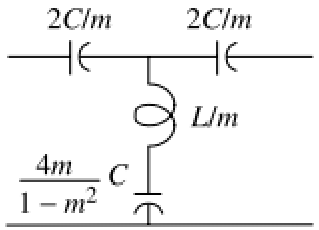
TABLE 9.3 Design Relations for Composite Filters

Low-Pass	High-Pass
Constant-k T-filter	*Constant-k T-filter*
 $Z_0 = \sqrt{L/C}$ $\omega_c = 2/\sqrt{LC}$ $L = 2Z_0/\omega_c$ $C = 2/Z_0\omega_c$	 $Z_0 = \sqrt{L/C}$ $\omega_c = 1/2\sqrt{LC}$ $L = 0.5Z_0/\omega_c$ $C = 0.5/Z_0\omega_c$
m-derived T-Section	*m-derived T-Section*
(Values of L and C are the same as above)	(Values of L and C are the same as above)
$m = \sqrt{1 - \left(\frac{f_c}{f_\infty}\right)^2}$	$m = \sqrt{1 - \left(\frac{f_\infty}{f_c}\right)^2}$
	

Input & Output Matching Sections

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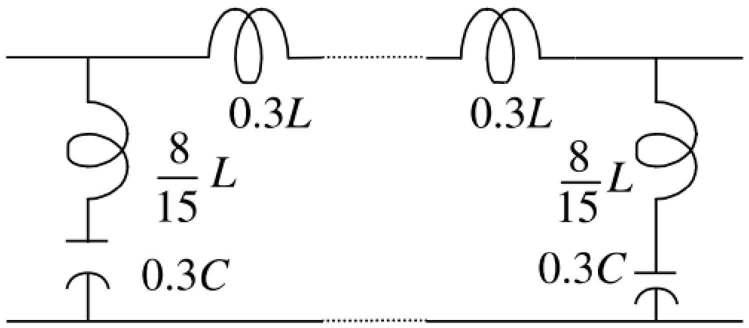
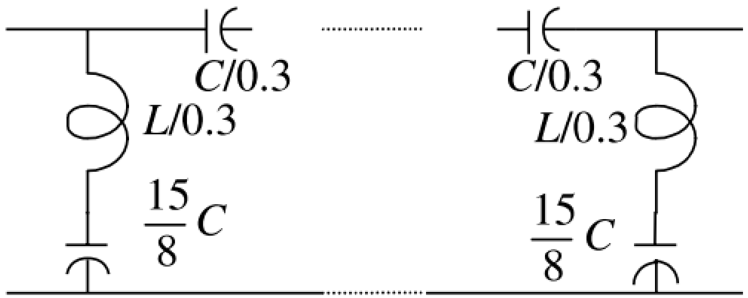
Low-Pass

Input and Output Matching Sections



High Pass

Input and Output Matching Sections



Composite Filters

❖ *Example*

- Design a high-pass composite filter with a nominal impedance of 75Ω. It must pass all signals over 2 MHz. Assume that $f_\infty = 1.95$ MHz.

❖ *Solution*

From Table 9.3 (slide 31), we find the components of its constant-*k* section as follows:

$$L = \frac{75}{2 \times 2 \times \pi \times 2 \times 10^6} \text{H} = 2.984\,\mu\text{H}$$

and

$$C = \frac{1}{2 \times 2 \times \pi \times 2 \times 10^6 \times 75} \text{F} = 530.5\,\text{pF}$$

Composite Filters

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Similarly, the component values for its m-derived filter section are determined as follows:

$$m = \sqrt{1 - \left(\frac{f_\infty}{f_c}\right)^2} = \sqrt{1 - \left(\frac{1.95}{2}\right)^2} = 0.222$$

$$\frac{2C}{m} = 4.775\,\text{nF}$$

$$\frac{L}{m} = 13.43\,\mu\text{H}$$

$$\frac{4m}{1-m^2}C = 0.496\,\text{nF}$$

Composite Filters

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The component values for the bisected π -section to be used at its input and output ports are found as

$$\frac{C}{0.3} = 1.768\,\text{nF}$$

$$\frac{L}{0.3} = 9.947\,\mu\text{H}$$

$$\frac{15}{8}C = 0.9947\,\text{nF}$$